



22. In the diagram, $\angle ABE = 10^{\circ}$; $\angle EBC = 70^{\circ}$; $\angle ACD = 50^{\circ}$; $\angle DCB = 20^{\circ}$; $\angle DEF = \alpha$.

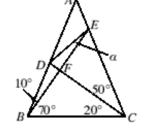
Which of the following is equal to $\tan \alpha$?

$$A \frac{\tan 10^{\circ} \tan 20^{\circ}}{\tan 50^{\circ}}$$

$$\frac{\tan 10^{\circ} \tan 50^{\circ}}{\tan 70^{\circ}}$$

$$B = \frac{\tan 10^{\circ} \tan 20^{\circ}}{\tan 70^{\circ}}$$

C



D $\frac{\tan 20^{\circ} \tan 50^{\circ}}{\tan 70^{\circ}}$

 $E = \frac{\tan 10^{\circ} \tan 70^{\circ}}{\tan 50^{\circ}}$

1192



©UKMT

As the sum of the angles in a triangle is 180° , in triangle CBF, $\angle BFC = 90^{\circ}$. As vertically opposite angles are equal $\angle DFE = \angle BFC = 90^{\circ}$. As the sum of the angles on a straight line is 180° , $\angle DFB = \angle EFC = 90^{\circ}$. Hence in triangle EFD, $\tan \alpha = \frac{DF}{EF}$: in triangle DFB, $\tan 10^{\circ} = \frac{DF}{FB}$: in triangle BFC, $\tan 20^{\circ} = \frac{FB}{FC}$ and in triangle CEF, $\tan 50^{\circ} = \frac{EF}{FC}$. Thus $\tan \alpha = \frac{DF}{EF} = \frac{\tan 10^{\circ} FB}{EF} = \frac{\tan 10^{\circ} \tan 20^{\circ}}{EF} = \frac{\tan 10^{\circ} \tan 20^{\circ}}{\tan 50^{\circ}}$.