



16. The diagram shows the ellipse whose equation is $x^2 + y^2 - xy + x - 4y = 12$. The curve cuts the y-axis at points A and C and cuts the x-axis at points B and D. What is the area of the inscribed quadrilateral ABCD?

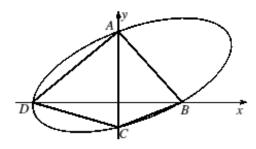
A 28

B 36

C 42

D 48

E 56



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16. A At points A and C, x = 0. So $y^2 - 4y = 12$, i.e. (y - 6)(y + 2) = 0, i.e. y = 6 or y = -2. So C is (0, -2) and A is (0, 6). At points B and D, y = 0. So $x^2 + x = 12$, i.e. (x - 3)(x + 4) = 0, i.e. x = 3 or x = -4. So D is (-4, 0) and B is (3, 0). Therefore the areas of triangles DAB and DBC are $\frac{1}{2} \times 7 \times 6 = 21$ and $\frac{1}{2} \times 7 \times 2 = 7$. So ABCD has area 28. {It is left to the reader to prove that area ABCD = $\frac{1}{2}BD \times AC$.}