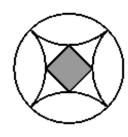




- 17. The diagram shows a pattern found on a floor tile in the cathedral in Spoleto, Umbria. A circle of radius 1 surrounds four quarter circles, also of radius 1, which enclose a square. The pattern has four axes of symmetry. What is the side length of the square?

  - A  $\frac{1}{\sqrt{2}}$  B  $2-\sqrt{2}$  C  $\frac{1}{\sqrt{3}}$  D  $\frac{1}{2}$  E  $\sqrt{2}-1$



1287



©UKMT

В In the diagram, B is the centre of the quarter-circle arc AC; D is the 17. point where the central square touches arc AC; F is the point where the central square touches arc CE; O is the centre of the circle.

As both the circle and arc AC have radius 1, OABC is a square of side 1. By Pythagoras' Theorem:  $OB^2 = 1^2 + 1^2$ . So  $OB = \sqrt{2}$ . Therefore  $OD = OB - DB = \sqrt{2} - 1$ . By a similar argument,  $OF = \sqrt{2} - 1$ . Now  $DF^2 = OD^2 + OF^2 = 2 \times OD^2$  since OD = OF. So the side of the square is  $\sqrt{2} \times OD = \sqrt{2}(\sqrt{2} - 1) = 2 - \sqrt{2}$ .

