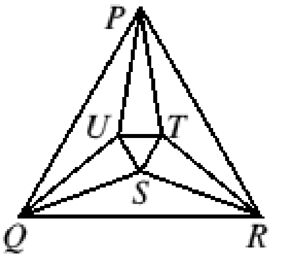
17. The equilateral triangle PQR has side-length 1. The lines PT and PU trisect the angle RPQ, the lines RS and RT trisect the angle QRP and the lines QS and QU trisect the angle PQR.

What is the side-length of the equilateral triangle STU?



A $\dfrac{\cos 80°}{\cos 20°}$ B $\frac{1}{3}\cos 20°$ C $\cos^2 20°$ D $\frac{1}{6}$ E $\cos 20° \cos 80°$

 ©UKMT

---

17. **A** Triangle PQR is equilateral so $\angle QPU = \angle UPT = \angle TPR = 20°$. Triangle PUT is isosceles, so $\angle PUT = 80°$. Let X be the midpoint of PQ and Y be the midpoint of UT.

Considering the right-angled triangle PXU gives $\cos 20° = \dfrac{PX}{PU} = \dfrac{\frac{1}{2}}{PU}$, so $PU = \dfrac{1}{2\cos 20°}$.

Considering the right-angled triangle PUY gives $\cos 80° = \dfrac{UY}{PU}$, so $UY = PU \cos 80° = \dfrac{\cos 80°}{2\cos 20°}$. Therefore $UT = 2UY = \dfrac{2\cos 80°}{2\cos 20°} = \dfrac{\cos 80°}{\cos 20°}$.

{*Note that triangle UTS is a Morley triangle, named after the mathematician Frank Morley. His 1899 trisector theorem states that in any triangle, the three points of intersection of the adjacent angle trisectors form an equilateral triangle, in this case, triangle UTS.*}