23. Which of the following have no real solutions?

(i) $2x < 2^x < x^2$ (ii) $x^2 < 2x < 2^x$ (iii) $2^x < x^2 < 2x$

(iv) $x^2 < 2^x < 2x$ (v) $2^x < 2x < x^2$ (vi) $2x < x^2 < 2^x$

A (i) and (iii) B (i) and (iv) C (ii) and (iv)

D (ii) and (v) E (iii) and (v)

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23. E If the graphs of $y = 2x$, $y = 2^x$ and $y = x^2$ are sketched on the same axes it can be seen that case (i) holds for $2 < x < 4$, case (ii) holds for $0 < x < 1$, case (iv) holds for $1 < x < 2$ and case (vi) holds for $x > 4$.

There are no real solutions for case (iii). Consider $x^2 < 2x$, which is true for $0 < x < 2$. However for $0 < x < 2$ it can be seen that $2^x > x^2$ rather than $2^x < x^2$ as stated.

There are no real solutions for case (v). Consider $2x < x^2$, which is true for $x < 0$ or $x > 2$. However, when $x < 0$ we have $2^x > 2x$ as 2^x is positive and $2x$ is negative, rather than $2^x < 2x$ as stated. Also, when $x = 2$ we have $2^x = 2x$, but for $x > 2$, $2^x > 2x$ rather than $2^x < 2x$ as stated.