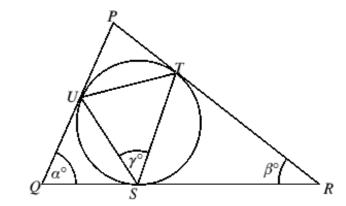




12. A circle touches the sides of triangle PQR at the points S, T and U as shown. Also $\angle PQR = \alpha^{\circ}$, $\angle PRQ = \beta^{\circ}$ and $\angle TSU = \gamma^{\circ}$. Which of the following gives γ in terms of α and β ?

A
$$\frac{1}{2}(\alpha + \beta)$$
 B $180 - \frac{1}{2}(\alpha + \beta)$
C $180 - (\alpha + \beta)D \alpha + \beta$
E $\frac{1}{3}(\alpha + \beta)$



1582



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12. A Each of the three sides of triangle PQR is a tangent to the circle. Two tangents to a circle which meet at a point are of equal length. So QU and QS are of equal length. Similarly RT = RS. This means that $\angle QUS = \angle QSU = \frac{1}{2}(180 - \alpha)$ and also $\angle RTS = \angle RST = \frac{1}{2}(180 - \beta)$. At S we can consider the sum of the three angles, so $\frac{1}{2}(180 - \alpha) + \gamma + \frac{1}{2}(180 - \beta) = 180$. Simplifying gives $90 - \frac{1}{2}\alpha + \gamma + 90 - \frac{1}{2}\beta = 180$ and so $\gamma = \frac{1}{2}(\alpha + \beta)$.