25. A function, defined on the set of positive integers, is such that $f(xy) = f(x) + f(y)$ for all x and y. It is known that $f(10) = 14$ and $f(40) = 20$. What is the value of $f(500)$?

A 29 B 30 C 39 D 48 E 50

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25. **C** Repeatedly using the rule that $f(xy) = f(x) + f(y)$ allows us to write $f(500)$ as $f(2 \times 2 \times 5 \times 5 \times 5) = f(2) + f(2) + f(5) + f(5) + f(5) = 2f(2) + 3f(5)$. We are given values for $f(40)$ and $f(10)$ and from them we need to calculate the values of $f(2)$ and $f(5)$. Now $f(40)$ can be written as $f(2) + f(2) + f(10)$ so $20 = 2f(2) + 14$ and therefore $f(2) = 3$. Similarly $f(10) = f(2) + f(5)$ so $14 = 3 + f(5)$ giving $f(5) = 11$. So $f(500) = 2f(2) + 3f(5) = 2 \times 3 + 3 \times 11 = 39$.