UKMT Averages Questions

(Answers follow after all the questions)

2005…

2. The number 2005 is the sum of a sequence of five consecutive positive integers. Which of the following numbers occurs in this sequence?

A 395 B 400 C 405 D 410 E 415

2007…

8. Travelling at an average speed of 100 km/hr, a train took 3 hours to travel to Birmingham. Unfortunately the train then waited just outside the station, which reduced the average speed for the whole journey to 90 km/hr. For how many minutes was the train waiting?

A 1 B 5 C 10 D 15 E 20

10. In 1954, a total of 6 527 mm of rain fell at Sprinkling Tarn and this set a UK record for annual rainfall. The tarn has a surface area of 23 450 m^2. Roughly how many million litres of water fell on Sprinkling Tarn in 1954?

A 15 B 150 C 1 500 D 15 000 E 150 000

2013…

10. Frank's teacher asks him to write down five integers such that the median is one more than the mean, and the mode is one greater than the median. Frank is also told that the median is 10. What is the smallest possible integer that he could include in his list?

A 3 B 4 C 5 D 6 E 7

2015…

5. The integer n is the mean of the three numbers 17, 23 and $2n$.
What is the sum of the digits of n?

A 4 B 5 C 6 D 7 E 8

2016…

11. In the grid below each of the blank squares and the square marked X are to be filled by the mean of the two numbers in its adjacent squares. Which number should go in the square marked X?

10			X		25

A 15 B 16 C 17 D 18 E 19

UKMT Averages Answers

2005...

2. **B** Let the five consecutive positive integers be $x-2, x-1, x, x+1, x+2$. Their sum is $5x$, so $5x = 2005$, that is $x = 401$. The five numbers are 399, 400, 401, 402, 403.

2007...

8. **E** The distance travelled to Birmingham by the train was 300 km. The time taken to travel this distance at an average speed of 90 km/hr is $\frac{300}{90}$ hr $= 3\frac{1}{3}$ hr $=$ 3 hr 20 min. So the train was waiting for 20 minutes.

10. **B** The volume of water which fell at Sprinkling Tarn in 1954 is approximately equal to $(25\,000 \times 6)$ m^3, that is 150 000 m^3. Now 1 m^3 $= 10^6$ cm^3 $= 10^6$ ml $=$ 1000 litres. So approximately 150 million litres of water fell on Sprinkling Tarn in 1954.

2013...

10. **B** The median is 10. Therefore the mode must be 11 and there must be two 11s in Frank's list. The mean is 9, so the total of the five numbers is 45. This means that the total of the two smallest integers is $45 - (10 + 2 \times 11) = 13$. The maximum size of the second largest integer is 9 so the smallest integer that Frank could include in his list is $13 - 9 = 4$.

2015...

5. **A** The mean of 17, 23 and $2n$ is given to be n, so $\dfrac{17 + 23 + 2n}{3} = n$ which gives $40 + 2n = 3n$. As n is then 40, the sum of the digits of n is 4.

2016...

11. **E** For each square to be filled with the mean of the numbers in the adjacent squares, the differences between all five pairs of adjacent numbers must be equal. This common difference is $\left(\dfrac{25 - 10}{5}\right)$ which is 3. The grid is then

10	13	16	**19**	22	25

and the 19 is in the desired square.