UKMT Geometry Questions

(Answers follow after all the questions)

2005…

6. How many differently shaped triangles exist in which no two sides are the same length, each side is of integral unit length and the perimeter of the triangle is less than 13 units?

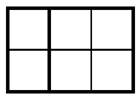
A 2 B 3 C 4 D 5 E 6

8. An examination paper is made by taking 5 large sheets of paper, folding the pile in half and stapling it. The pages are then numbered in order from 1 to 20. What is the sum of the three page numbers that are on the same sheet of paper as page number 5?

A 13 B 21 C 33 D 37 E 41

2007…

15. How many hexagons can be found in the diagram on the right if each side of a hexagon must consist of all or part of one of the straight lines in the diagram?



A 4 B 8 C 12 D 16 E 20

20. A triangle is cut from the corner of a rectangle. The resulting pentagon has sides of length 8, 10, 13, 15 and 20 units, though not necessarily in that order. What is the area of the pentagon?

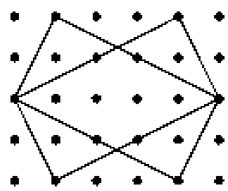
A 252.5 B 260 C 270 D 275.5 E 282.5

23. The sum of the lengths of the 12 edges of a cuboid is x cm. The distance from one corner of the cuboid to the furthest corner is y cm. What, in cm^2, is the total surface area of the cuboid?

A $\frac{x^2 - 2y^2}{2}$ B $x^2 + y^2$ C $\frac{x^2 - 4y^2}{4}$ D $\frac{xy}{6}$ E $\frac{x^2 - 16y^2}{16}$

2008…

11. The distance between two neighbouring dots in the dot lattice is 1 unit. What, in square units, is the area of the region where the two rectangles overlap?



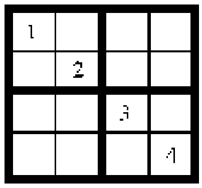
A 6 B $6\frac{1}{4}$ C $6\frac{1}{2}$ D 7 E $7\frac{1}{2}$

25. What is the area of the polygon formed by all points (x, y) in the plane satisfying the inequality $||x| - 2| + ||y| - 2| \leqslant 4$?

A 24 B 32 C 64 D 96 E 112

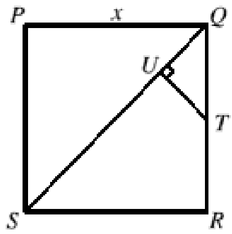
2009…

7. A mini-sudoku is a 4 by 4 grid, where each row, column and 2 by 2 outlined block contains the digits 1, 2, 3 and 4 once and once only. How many different ways are there of completing the mini-sudoku shown?



A 1 B 2 C 4 D 8 E 12

9. A square $PQRS$ has sides of length x. T is the midpoint of QR and U is the foot of the perpendicular from T to QS. What is the length of TU?

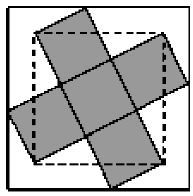


A $\frac{x}{2}$ B $\frac{x}{3}$ C $\frac{x}{\sqrt{2}}$ D $\frac{x}{2\sqrt{2}}$ E $\frac{x}{4}$

22. M and N are the midpoints of sides GH and FG, respectively, of parallelogram $EFGH$. The area of triangle ENM is $12\ \text{cm}^2$. What is the area of the parallelogram $EFGH$?

A $20\,\text{cm}^2$ B $24\,\text{cm}^2$ C $32\,\text{cm}^2$ D $48\,\text{cm}^2$ E more information is required

24. A figure in the shape of a cross is made from five 1×1 squares, as shown. The cross is inscribed in a large square whose sides are parallel to the dashed square, formed by four of the vertices of the cross. What is the area of the large outer square?



A 9 B $\frac{49}{5}$ C 10 D $\frac{81}{8}$ E $\frac{32}{3}$

2010…

5. A notice on Morecambe promenade reads: 'It would take 20 million years to fill Morecambe Bay from a bath tap.' Assuming that the flow from the bath tap is 6 litres a minute, what does the notice imply is the approximate capacity of Morecambe Bay in litres?

A 6×10^{10} B 6×10^{11} C 6×10^{12} D 6×10^{13} E 6×10^{14}

6. Dean runs up a mountain road at 8 km per hour. It takes him one hour to get to the top. He runs down the same road at 12 km per hour. How many minutes does it take him to run down the mountain?

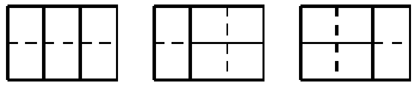
A 30 B 40 C 45 D 50 E 90

10. A square is cut into 37 squares of which 36 have area $1\ \text{cm}^2$. What is the length of the side of the *original* square?

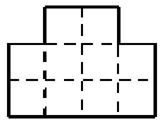
A 6 cm B 7 cm C 8 cm D 9 cm E 10 cm

2011…

8. A 2×3 grid of squares can be divided into 1×2 rectangles in three different ways.

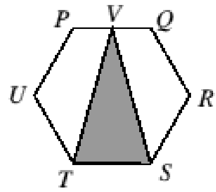


How many ways are there of dividing the bottom shape into 1×2 rectangles?



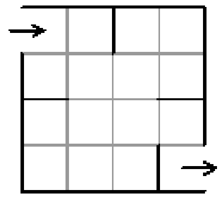
A 1 B 4 C 6 D 7 E 8

11. $PQRSTU$ is a regular hexagon and V is the midpoint of PQ. What fraction of the area of $PQRSTU$ is the area of triangle STV?



A $\frac{1}{4}$ B $\frac{2}{15}$ C $\frac{1}{3}$ D $\frac{2}{5}$ E $\frac{5}{12}$

13. The diagram represents a maze. Given that you can only move horizontally and vertically and are not allowed to revisit a square, how many different routes are there through the maze?



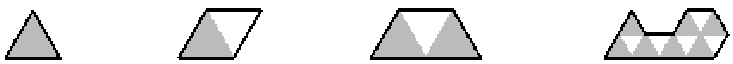
A 16 B 12 C 10 D 8 E 6

14. An equilateral triangle of side length 4 cm is divided into smaller equilateral triangles, all of which have side length equal to a whole number of centimetres. Which of the following cannot be the number of smaller triangles obtained?

A 4 B 8 C 12 D 13 E 16

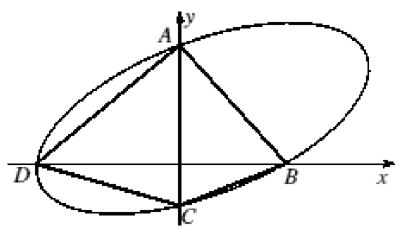
2012…

8. The diagrams below show four types of tile, each of which is made up of one or more equilateral triangles. For how many of these types of tile can we place three identical copies of the tile together, without gaps or overlaps, to make an equilateral triangle?



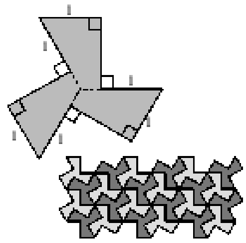
A 0 B 1 C 2 D 3 E 4

16. The diagram shows the ellipse whose equation is $x^2 + y^2 - xy + x - 4y = 12$. The curve cuts the y-axis at points A and C and cuts the x-axis at points B and D. What is the area of the inscribed quadrilateral $ABCD$?



A 28 B 36 C 42 D 48 E 56

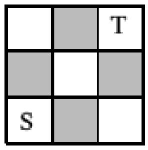
24. The top diagram on the right shows a shape that tiles the plane, as shown in the lower diagram. The tile has nine sides, six of which have length 1. It may be divided into three congruent quadrilaterals as shown. What is the area of the tile?



A $\frac{1+2\sqrt{3}}{2}$ B $\frac{4\sqrt{3}}{3}$ C $\sqrt{6}$D $\frac{3+4\sqrt{3}}{4}$ E $\frac{3\sqrt{3}}{2}$

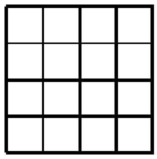
2013…

4. A route on the 3 × 3 board shown consists of a number of steps. Each step is from one square to an adjacent square of a different colour. How many different routes are there from square S to square T which pass through every other square exactly once?



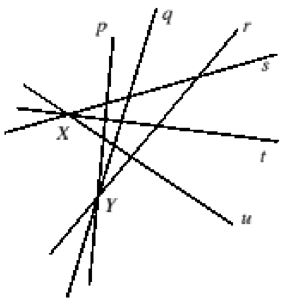
A 0 B 1 C 2 D 3 E 4

19. The 16 small squares shown in the diagram each have a side length of 1 unit. How many pairs of vertices are there in the diagram whose distance apart is an integer number of units?



A 40 B 64 C 108 D 132 E 16

25.



Challengeborough's underground train network consists of six lines, p, q, r, s, t, u, as shown. Wherever two lines meet there is a station which enables passengers to change lines. On each line, each train stops at every station.

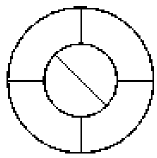
Jessica wants to travel from station X to station Y. She does not want to use any line more than once, nor return to station X after leaving it, nor leave station Y having reached it.

How many different routes, satisfying these conditions, can she choose?

A 9 B 36 C 41 D 81 E 720

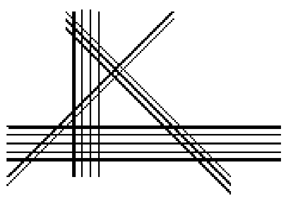
2014…

2. The diagram shows 6 regions. Each of the regions is to be painted a single colour, so that no two regions sharing an edge have the same colour. What is the smallest number of colours required?



A 2 B 3 C 4 D 5 E 6

8. The diagram shows four sets of parallel lines, containing 2, 3, 4 and 5 lines respectively.
How many points of intersection are there?

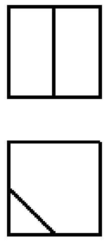


A 54 B 63 C 71 D 95 E 196

10. A rectangle has area $120\,\text{cm}^2$ and perimeter $46\,\text{cm}$. Which of the following is the length of each of the diagonals?

A 15cm B 16 cm C 17cm D 18 cm E 19 cm

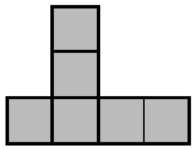
18. Beatrix decorates the faces of a cube, whose edges have length 2. For each face, she either leaves it blank, or draws a single straight line on it. Every line drawn joins the midpoints of two edges, either opposite or adjacent, as shown.



What is the length of the longest unbroken line that Beatrix can draw on the cube?

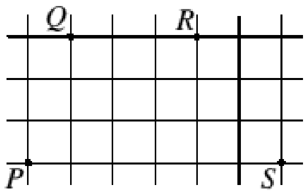
A 8 B $4+4\sqrt{2}$ C $6+3\sqrt{2}$ D $8+2\sqrt{2}$ E 12

20. The diagram shows six squares with sides of length 2 placed edge-to-edge. What is the radius of the smallest circle containing all six squares?



A $2\sqrt{5}$ B $2\sqrt{6}$ C 5 D $\sqrt{26}$ E $2\sqrt{7}$

21. Fiona wants to draw a 2-dimensional shape whose perimeter passes through all of the points P, Q, R and S on the grid of squares shown. Which of the following can she draw?



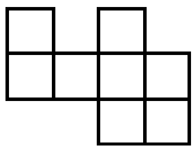
(i) A circle (ii) An equilateral triangle

(iii) A square

A only (i) and (ii) B only (ii) and (iii) C only (i) and (iii) D all of (i), (ii) and (iii) E none of (i), (ii) and (iii)

2015…

8. The diagram shows eight small squares. Six of these squares are to be shaded so that the shaded squares form the net of a cube.



In how many different ways can this be done?

A 10 B 8 C 7 D 6 E 4

9. Four different straight lines are drawn on a flat piece of paper. The number of points where two or more lines intersect is counted.

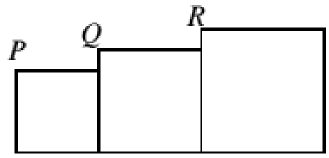
Which of the following could **not** be the number of such points?

A 1 B 2 C 3 D 4 E 5

11. Rahid has a large number of cubic building blocks. Each block has sides of length 4 cm, 6 cm or 10 cm. Rahid makes little towers built from three blocks stacked on top of each other. How many different heights of tower can he make?

A 6 B 8 C 9 D 12 E 27

19. Three squares are arranged as shown so that their bases lie on a straight line. Also, the corners P, Q and R lie on a straight line. The middle square has sides that are 8 cm longer than the sides of the smallest square. The largest square has sides of length 50 cm.



There are two possible values for the length (in cm) of the sides of the smallest square. Which of the following are they?

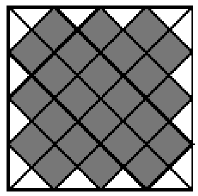
A 2, 32 B 4, 42 C 4, 34 D 32, 40 E 34, 42

20. A square ink pad has sides of length 1 cm. It is covered in black ink and carefully placed in the middle of a piece of white paper. The square pad is then rotated 180° about one of its corners so that all of the pad remains in contact with the paper throughout the turn. The pad is then removed from the paper. What area of paper, in cm^2, is coloured black?

A $\pi + 2$ B $2\pi - 1$ C 4 D $2\pi - 2$ E $\pi + 1$

2016…

8. Points are drawn on the sides of a square, dividing each side into n equal parts (so, in the example shown, $n = 4$).
The points are joined in the manner indicated, to form several small squares (24 in the example, shown shaded) and some triangles.
How many small squares are formed when $n = 7$?

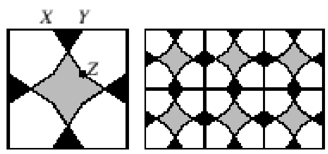


A 56 B 84 C 140 D 840 E 5040

19. Let S be a set of five different positive integers, the largest of which is m. It is impossible to construct a quadrilateral with non-zero area, whose side-lengths are all distinct elements of S. What is the smallest possible value of m?

A 2 B 4 C 9 D 11 E 12

20. Michael was walking in Marrakesh when he saw a tiling formed by tessellating the square tile as shown.
The tile has four lines of symmetry and the length of each side is 8 cm. The length of XY is 2 cm. The point Z is such that XZ is a straight line and YZ is parallel to sides of the square.



What is the area of the central grey octagon?

A 6 cm^2 B 7 cm^2 C 8 cm^2 D 9 cm^2 E 10 cm^2

UKMT Geometry Answers

2005…

6. **B** The longest side of any triangle is shorter than the sum of the lengths of the other two sides. This condition means that the only possible triangles having different sides of integral unit length, and having perimeters less than 13 units, have sides of length 2, 3, 4 or 2, 4, 5 or 3, 4, 5.

8. **D** The first large sheet of paper will hold pages 1, 2, 19 and 20; the second will hold pages 3, 4, 17 and 18; the third will hold pages 5, 6, 15 and 16.

2007…

15. **D** In the given diagram, there are four hexagons congruent to the hexagon in Figure (i), four hexagons congruent to the hexagon in Figure (ii) and eight hexagons congruent to the hexagon in Figure (iii).

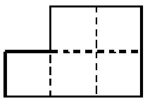


Figure (i)

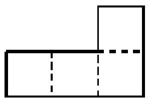


Figure (ii)

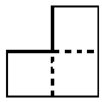
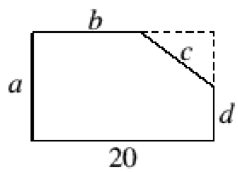


Figure (iii)

20. **C** The diagram shows the original rectangle with the corner cut from it to form a pentagon. It may be deduced that the length of the original rectangle is 20 and that a, b, c, d are 8, 10, 13, 15 in some order.



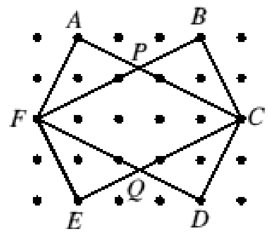
By Pythagoras' Theorem $c^2 = (20 - b)^2 + (a - d)^2$. So c cannot be 8 as there is no right-angled triangle having integer sides and hypotenuse 8. If $c = 10$, then $(20 - b)$ and $(a - d)$ are 6 and 8 in some order, but this is not possible using values of 8, 13 and 15. Similarly, if $c = 15$, then $(20 - b)$ and $(a - d)$ are 9 and 12 in some order, but this is not possible using values of 8, 10 and 13. However, if $c = 13$, then $(20 - b)$ and $(a - d)$ are 5 and 12 in some order, which is true if and only if $a = 15$, $b = 8$, $d = 10$.
So the area of the pentagon is $20 \times 15 - \frac{1}{2} \times 5 \times 12 = 270$.

23. **E** Let the lengths of the sides of the cuboid, in cm, be a, b and c. So $4(a + b + c) = x$. Also, by Pythagoras' Theorem $a^2 + b^2 + c^2 = y^2$. Now the total surface area of the cuboid is

$$2ab + 2bc + 2ca = (a + b + c)^2 - (a^2 + b^2 + c^2) = \left(\frac{x}{4}\right)^2 - y^2 = \frac{x^2 - 16y^2}{16}.$$

2008…

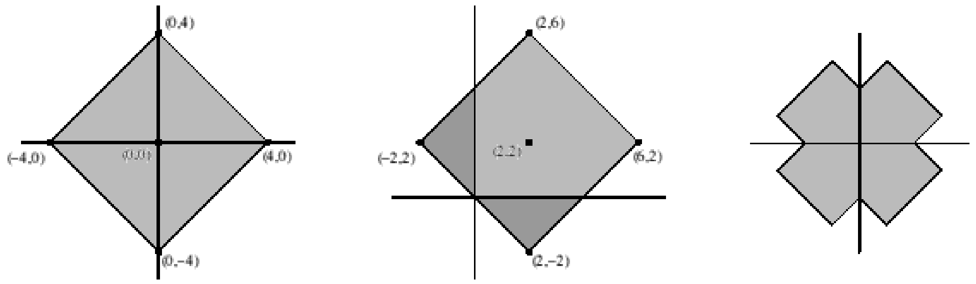
11. B Let the six points where lines meet on the dot lattice be A, B, C, D, E, F as shown and let the other two points of intersection be P (where AC and BF meet) and Q (where CE and DF meet).



Triangles APB and CPF are similar with base lengths in the ratio 3:5. Hence triangle CPF has height $\frac{5}{8} \times 2 = \frac{5}{4}$ units and base length 5 units so that its area is $\frac{1}{2} \times \frac{5}{4} \times 5$ square units. Since the same is true of triangle CQF, the required area is $\frac{5}{4} \times 5 = 6\frac{1}{4}$ square units.

25. D To work out the area of $\big||x|-2\big| + \big||y|-2\big| \leqslant 4$, we first consider the region $|x| + |y| \leqslant 4$ which is shown in (a). This region is then translated to give $|x - 2| + |y - 2| \leqslant 4$ as shown in (b).

By properties of the modulus, if the point (x, y) lies in the polygon, then so do $(x, -y)$, $(-x, y)$ and $(-x, -y)$. Thus $\big||x|-2\big| + \big||y|-2\big| \leqslant 4$ can be obtained from (b) by reflecting in the axes and the origin, as shown in (c).



(a) (b) (c)

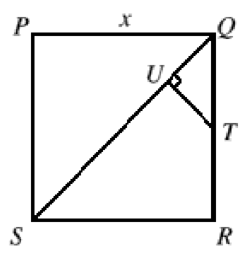
Hence the required area is 4 times the area in the first quadrant. From (b), the required area in the first quadrant is the area of a square of side $4\sqrt{2}$ minus two triangles (cut off by the axes) which, combined, make up a square of side $2\sqrt{2}$. So the area in the first quadrant is $(4\sqrt{2})^2 - (2\sqrt{2})^2 = 32 - 8 = 24$.

Hence the area of the polygon is $4 \times 24 = 96$ square units.

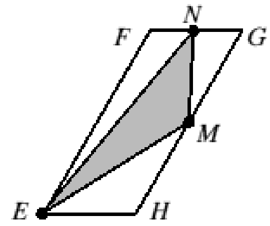
2009…

7. B The top left 2 by 2 outlined block must contain a 3 and a 4 and this can be done in two ways. For each choice there is only one way to complete the entire mini-sudoku.

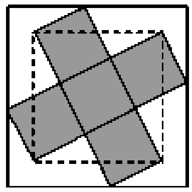
9. D As T is the midpoint of QR then $QT = \frac{1}{2}x$.
Since $\angle UQT = \angle SQR = 45°$ and $\angle QUT = 90°$, $\angle UTQ = 45°$.
Thus triangle QTU is isosceles with $UQ = UT$.
In triangle QTU, by Pythagoras' Theorem, $QT^2 = QU^2 + TU^2$.
Hence $(\frac{1}{2}x)^2 = 2TU^2$ so $TU^2 = \frac{1}{8}x^2$ giving $TU = \dfrac{x}{2\sqrt{2}}$.



22. **C** Let the perpendicular distance between EH and FG be x cm and the area of the parallelogram $EFGH$ be y cm^2. Thus $y = FG \times x$.
The area of triangle EFN is $\frac{1}{2}FN \times x = \frac{1}{2} \times \frac{1}{2} \times FG \times x = \frac{1}{4}y$ cm^2.
Likewise the areas of triangles EHM and NGM are $\frac{1}{4}y$ cm^2 and $\frac{1}{8}y$ cm^2 respectively.
The area of triangle ENM is 12 cm^2, hence $y = 12 + \frac{5}{8}y$ and so $y = 32$. Hence the area of the parallelogram $EFGH$ is 32 cm^2.



24. **B** A shaded triangle is congruent to an unshaded triangle (ASA).
Hence the area of the dashed square is equal to the area of the cross and both are 5.
Thus the side-length of the dashed square is $\sqrt{5}$.
Hence the sides of a shaded triangle are: $\frac{1}{2}$, 1 and $\frac{1}{2}\sqrt{5}$.
Now the perpendicular distance between the squares is equal to the altitude, h, of the shaded triangle. The area of such a triangle is
$\frac{1}{2} \times (\frac{1}{2} \times 1) = \frac{1}{4}$ so that $\frac{1}{2} \times (\frac{1}{2}\sqrt{5} \times h) = \frac{1}{4}$ which gives $h = \dfrac{1}{\sqrt{5}}$.
Hence the length of the sides of the outer square are $\sqrt{5} + 2 \times \dfrac{1}{\sqrt{5}} = \dfrac{5}{\sqrt{5}} + \dfrac{2}{\sqrt{5}} = \dfrac{7}{\sqrt{5}}$.
Thus the area of the large square is $\left(\dfrac{7}{\sqrt{5}}\right)^2 = \dfrac{49}{5}$.



2010…

5. **D** If the statement is true then the capacity (in litres) of Morecambe Bay is approximately:

$$20 \times 10^6 \times 365 \times 24 \times 60 \times 6 = 10^8 \times (6 \times 365) \times (2 \times 24) \times 6$$
$$\approx 6 \times 10^8 \times 2000 \times 50 = 6 \times 10^{13}.$$

6. **B** The length of the road is 8km. Hence the time taken to run down the mountain is $\frac{8}{12}$ hours $= \frac{8}{12} \times 60$ min $= 40$ min.

10. **E** Let the original square have sides of length y cm and the single square which is not 1×1 have sides of length x cm. Then $y^2 = 36 + x^2$, and so $y^2 - x^2 = 36$ and hence $(y + x)(y - x) = 36$.
As $36 = 2^2 \times 3^2$ and $y + x > y - x$ the possible factors of 36 are:

$y + x$	$y - x$	y	x	
9	4	$6\frac{1}{2}$	$2\frac{1}{2}$	impossible
12	3	$7\frac{1}{2}$	$4\frac{1}{2}$	impossible
18	2	10	8	possible
36	1	$18\frac{1}{2}$	$17\frac{1}{2}$	impossible

We can check that $10^2 = 36 + 8^2 = 100$ and hence the length of the side of the *original* square is 10 cm.

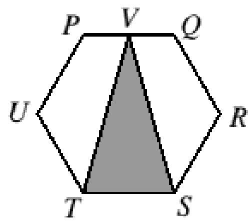
2011…

8. **C** The 1×2 rectangles can appear in two different ways: A  or B .
If, in the given shape, A forms the top 1×2 rectangle then the possible different ways to fill the remaining 2×4 rectangle are, from left to right:

A, A, A, A; A, A, B, B; B, A, A, B; B, B, A, A; B, B, B, B.

If A does not form the top 1×2 rectangle, then the only possible way is to use 4B and an A. Hence there are 6 ways of dividing the given shape into 1×2 rectangles.

11. **C** Let x be the side length of the regular hexagon $PQRSTU$ and let $h = PT = QS$, the perpendicular height of triangle STV.
Thus the area of triangle STV is $\frac{1}{2}xh$ and the areas of triangles PTV and QSV are both $\frac{1}{2}\left(\frac{1}{2}xh\right) = \frac{1}{4}xh$. The perpendicular heights of triangles PTU and QRS are



$$\frac{UR - PQ}{2} = \frac{2x - x}{2} = \frac{x}{2}.$$

Hence the area of each of triangles PTU and QRS is $\frac{1}{2}h \times \frac{1}{2}x = \frac{1}{4}hx$.
Therefore the area of triangle STV is one third of the area of $PQRSTU$.

13. **D** Let the centres of the starting and finishing squares in the maze have coordinates (1,4) and (4,1) respectively. Each path must pass through (2,3) and (3,2). There are two different routes from (1,4) to (2,3). The next visit is to (3,3) or (2,2).
When visiting (3,3) the next visit has to be (3,2) as (3,4), (4,3) and (4,4) cannot be visited without subsequently revisiting a square. From (2,2) the next valid visit is to (1,2), (2,1) or (3,2). From each of these points there is only one route to (3,2). Thus there are four ways of visiting (3,2). Upon visiting (3,2), the only valid route through the maze is (4,2) then (4,1).
Hence the number of different routes through the maze is $2 \times 4 = 8$.

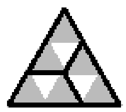
14. **C** Let us define T_n to represent an equilateral triangle with side length n cm. Then an equilateral triangle of side length 4 cm can be divided into smaller equilateral triangles as follows:

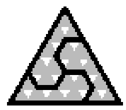
$1 \times T_3$ and $7 \times T_1$	$4 \times T_2$	$3 \times T_2$ and $4 \times T_1$
$2 \times T_2$ and $8 \times T_1$	$1 \times T_2$ and $12 \times T_1$	$16 \times T_1$.

The number of triangles used are: 8, 4, 7, 10, 13 and 16. So it is not possible to dissect the original triangle into 12 triangles.

2012…

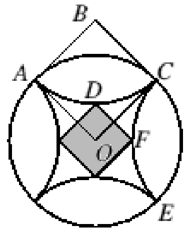
8. **C** If an equilateral triangle is split into a number of smaller identical equilateral triangles then there must be one small triangle in the top row, three small triangles in the row below, five small triangles in the row below that and so on. So the total number of small triangles is 4 or 9 or 16 etc. These are all squares and it is left to the reader to prove that the sum of the first n odd numbers is n^2. So, for three copies of a given tile to form an equilateral triangle, the number of triangles which comprise the tile must be one third of a square number.





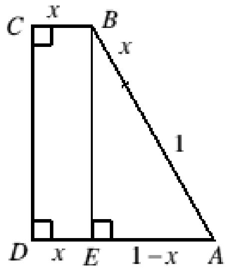
Only the tiles made up of three equilateral triangles and twelve equilateral triangles satisfy this condition. However, it is still necessary to show that three copies of these tiles can indeed make equilateral triangles. The diagrams above show how they can do this.

17. B In the diagram, B is the centre of the quarter-circle arc AC; D is the point where the central square touches arc AC; F is the point where the central square touches arc CE; O is the centre of the circle.



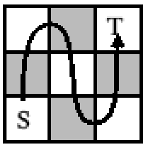
As both the circle and arc AC have radius 1, $OABC$ is a square of side 1. By Pythagoras' Theorem: $OB^2 = 1^2 + 1^2$. So $OB = \sqrt{2}$. Therefore $OD = OB - DB = \sqrt{2} - 1$. By a similar argument, $OF = \sqrt{2} - 1$. Now $DF^2 = OD^2 + OF^2 = 2 \times OD^2$ since $OD = OF$. So the side of the square is $\sqrt{2} \times OD = \sqrt{2}(\sqrt{2} - 1) = 2 - \sqrt{2}$.

24. B The diagram shows one of the three quadrilaterals making up the tile, labelled and with a line BE inserted. Note that it is a trapezium. As three quadrilaterals fit together, it may be deduced that $\angle ABC = 360° \div 3 = 120°$, so $\angle BAD = 60°$. It may also be deduced that the length of AB is $1 + x$, where x is the length of BC. Now $\cos\angle BAD = \cos 60° = \frac{1}{2} = \frac{1-x}{1+x}$. So $1 + x = 2 - 2x$, i.e. $x = \frac{1}{3}$. The area of $ABCD$ is $\frac{1}{2}(AD + BC) \times CD = \frac{1}{2}\left(1 + \frac{1}{3}\right) \times \frac{4}{3}\sin 60° = \frac{2}{3} \times \frac{4}{3} \times \frac{\sqrt{3}}{2} = \frac{4\sqrt{3}}{9}$. So the area of the tile is $3 \times \frac{4\sqrt{3}}{9} = \frac{4\sqrt{3}}{3}$.

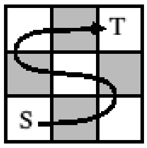


2013…

4. C In order to pass through each square exactly once, a route must pass in and out of both unlabelled corner squares and also pass through the middle. Passing in and out of a corner involves three squares, coloured grey, white and grey in that order. Passing in and out of the two unlabelled corners therefore accounts for six unlabelled squares, leaving only the middle square which must be in the middle of any possible route. So, there are two possible routes as shown.



or



19. C There are 25 vertices in the diagram. Each vertex is part of a row of 5 vertices and a column of 5 vertices. Each vertex is therefore an integer number of units away from the 4 other vertices in its row and from the other 4 vertices in its column. This appears to give $25 \times (4 + 4) = 200$ pairs. However, counting in this manner includes each pair twice so there are only 100 different pairs.
By using the Pythagorean triple 3, 4, 5, each corner vertex is five units away from two other non-corner vertices, giving another 8 pairs. No other Pythagorean triples include small enough numbers to yield pairs of vertices on this grid.
Thus the total number of pairs is 108.

25. **D** Jessica must travel alternately on lines which are connected to station X (i.e. s, t or u), and connected to station Y (i.e. p, q or r). In order to depart from X and end her journey at Y, she must travel along an even number of lines. This can be 2, 4 or 6 lines, making 1, 3 or 5 changes respectively.

Case A, 2 lines: Jessica leaves station X along one of the lines s, t or u, makes one change onto one of lines p, q or r and reaches station Y. Here there are 3×3 possibilities.

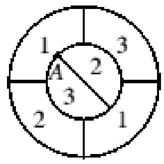
Case B, 4 lines: Jessica leaves station X along one of the lines s, t or u and makes her first change onto one of lines p, q or r. She then makes her second change onto either of the two lines s, t or u on which she has not previously travelled and her third change onto either of the two lines p, q or r on which she has not previously travelled and reaches station Y. Here there are $3 \times 3 \times 2 \times 2$ possibilities.

Case C, 6 lines: Her journey is as described in Case B but her fourth change is onto the last of the lines s, t or u on which she has not previously travelled and her fifth change is onto the last of the lines p, q or r on which she has not previously travelled. Here there are $3 \times 3 \times 2 \times 2 \times 1 \times 1$ possibilities.

So in total Jessica can choose $9 + 36 + 36 = 81$ different routes.

2014…

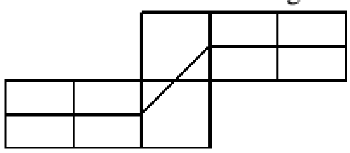
2. **B** There are points on the diagram, such as A, where the edges of three regions meet, so three or more different colours are required.
A colouring with three colours is possible as shown, so the smallest number of colours required is three.



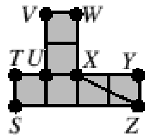
8. **C** The intersections occur in six groups and the total number of points is $2 \times 3 + 2 \times 4 + 2 \times 5 + 3 \times 4 + 3 \times 5 + 4 \times 5$ which is $6 + 8 + 10 + 12 + 15 + 20 = 71$.

10. **C** Let the length of the rectangle be x cm and its width be y cm. The area is given as $120\ \text{cm}^2$ so $xy = 120$. The perimeter is 46 cm, so $46 = 2x + 2y$ and therefore $23 = x + y$. Using Pythagoras' Theorem, the length of the diagonal is $\sqrt{x^2 + y^2}$. As $x^2 + y^2 = (x + y)^2 - 2xy$, $\sqrt{x^2 + y^2} = \sqrt{23^2 - 2 \times 120} = \sqrt{529 - 240} = \sqrt{289} = 17$. So the diagonal has length 17 cm.

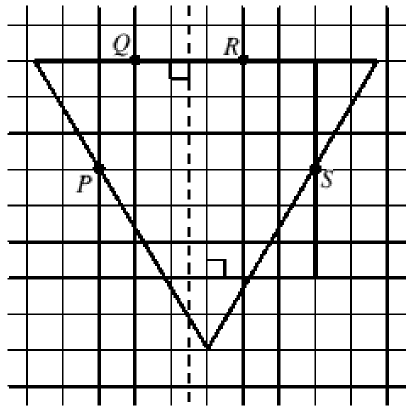
18. **D** To draw the longest unbroken line Beatrix must be able to draw her design on the net of a cube without taking her pen off the paper. She must minimise the number of lines of length $\sqrt{2}$ and maximise the number of lines of length 2. If no lines of length $\sqrt{2}$ are used, the maximum number of lines of length 2 is four, forming a loop and leaving two faces blank. Thus the longest possible unbroken line would have four lines of length 2 and two lines of length $\sqrt{2}$. A possible configuration to achieve this is shown in the diagram. The length of Beatrix's line is then $8 + 2\sqrt{2}$. Note: This path is not a loop but it is not required to be.



20. **A** It is always possible to draw a circle through three points which are not on a straight line. The smallest circle containing all six squares must pass through (at least) three of the eight vertices of the diagram. Of all such circles, the smallest passes through S, V and Z and has its centre at X. The radius is then $\sqrt{4^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$.

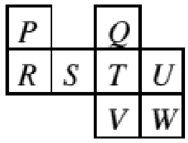


21. **B** The diagram shows that it is possible to draw a square whose edges go through P, Q, R and S. By drawing lines through P and S each making an angle of 60° with QR, we can construct an equilateral triangle, as shown, whose edges pass through P, Q, R and S. However there is no circle through these four points. The centre of such a circle would be equidistant from Q and R, and hence would lie on the perpendicular bisector of QR. Similarly it would lie on the perpendicular bisector of PS, but these perpendicular bisectors are parallel lines which don't meet.

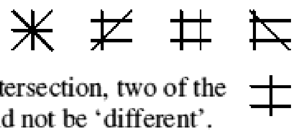


2015…

8. **D** Let the squares in the diagram be labelled as shown. Each of the nets formed from six squares must contain all of R, S and T. The net must also include one of P and Q (but not both as they will fold into the same position), and any two of U, V and W. This therefore gives $2 \times 3 = 6$ different ways.



9. **B** Possible configurations of four different straight lines drawn in a plane are shown here to give 1, 3, 4 and 5 points of intersection respectively. In order to have exactly 2 points of intersection, two of the straight lines would need to lie in the same position and so would not be ‘different’.

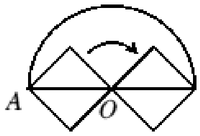


11. **C** There are several different ways to count systematically the number of towers that Rahid can build. Here is one way.

	All blocks the same size			Exactly two blocks the same size						All blocks of different sizes
	10 10 10	6 6 6	4 4 4	4 10 10	6 10 10	4 6 6	10 6 6	6 4 4	10 4 4	4 6 10
Total height	30	18	12	24	26	16	22	14	18	20

So there are nine different heights of tower (as the height of 18cm can be made from $6 + 6 + 6$ or $10 + 4 + 4$).

19. A Let the length of the side of the smallest square be x cm. So the three squares have sides of lengths x cm, $(x + 8)$ cm and 50 cm respectively. The gradient of PQ is then $\frac{8}{x}$ and the gradient of PR is $\frac{50 - x}{x + x + 8}$. As P, Q and R lie on a straight line, $\frac{8}{x} = \frac{50 - x}{2x + 8}$ so $8(2x + 8) = x(50 - x)$. Expanding gives $16x + 64 = 50x - x^2$ and therefore $x^2 - 34x + 64 = 0$, giving $x = 2$ or 32.

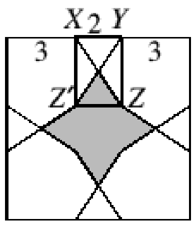
20. E 

Let the corner of the square about which it is rotated be O and the opposite vertex of the square be A. As the circle is rotated through $180°$ about O, the vertex A travels along a semicircle whose centre is O. The area coloured black by the ink is then formed from two half squares and a semicircle. The square has side-length 1, so $OA = \sqrt{2}$. The total area of the two half squares and the semicircle is $2 \times (\frac{1}{2} \times 1 \times 1) + \frac{1}{2} \times \pi \times (\sqrt{2})^2$ which is $1 + \pi$.

2016…

8. B One way to count the number of small squares formed is to divide the large square into four quarters along its two diagonals. The number of small squares formed is $4 \times T_{n-1}$, where T_{n-1} is the $(n - 1)$th triangular number. When $n = 7$, this is $4 \times \frac{1}{2}(6 \times 7)$ which is 4×21. So 84 squares are formed.

19. D Let S consist of h, j, k, l, m in ascending order of size. We want m to be as small as possible. Given three side-lengths, there is a quadrilateral with non-zero area with a specified fourth side-length if and only if the fourth side-length is less than the sum of the other three side-lengths. To ensure that j, k, l, m are not the side-lengths of such a quadrilateral, we must have $m \geqslant j + k + l$. Likewise, considering h, j, k, l, we must have $l \geqslant h + j + k$. Since the smallest possible values of h, j and k are 1, 2 and 3 respectively then $l \geqslant 1 + 2 + 3$ so 6 is the smallest value of l. Also $m \geqslant 2 + 3 + 6$ so 11 is the smallest value of m.

20. E 

Let the point Z' be directly below X, so that $XYZZ'$ is a rectangle. As the length of XY is 2 cm, the distance from Y to the nearest corner of the square is 3 cm. The area of $XYZZ'$ is 2 cm × 3 cm which is 6 cm^2. The diagonals XZ and YZ' split $XYZZ'$ into quarters and each has area $1\frac{1}{2}$ cm^2. The central grey octagon is formed from a square with side $Z'Z$ of length 2 cm together with four triangles, each of area $1\frac{1}{2}$ cm^2. The total area of the shaded octagon is $2 \times 2 + 4 \times 1\frac{1}{2}$ which is 10 cm^2.