UKMT Numberwork Questions
(Answers follow after all the questions)

2005…

1. What is the value of 2005 plus 2005 thousandths?

A 2005.002005 B 2005.2005 C 2007.005 D 2025.05 E 2205.5

5. Last year Rachel took part in a swimathon. Every day for 9 weeks she swam the same number of lengths, either in a 25m indoor pool or a 20m outdoor pool. Later she discovered that she had swum the same total distance in each pool. On how many days did Rachel swim in the indoor pool?

A 45 B 42 C 35 D 32 E 28

7. Consider the arithmetic sequences 1998, 2005, 2012, … and 1996, 2005, 2014, …. Which is the next number after 2005 that appears in both sequences?

A 2054 B 2059 C 2061 D 2063 E 2068

12. The positive integer x is a multiple of 7, and $\sqrt{x}$ is between 15 and 16. What is the number of possible values of x?

A 1 B 2 C 3 D 4 E 5

14. A square number is divided by 6. Which of the following could not be the remainder?

A 0 B 1 C 2 D 3 E 4

22. Given that $S = (x + 20) + (x + 21) + (x + 22) + \ldots + (x + 100)$, where x is a positive integer, what is the smallest value of x such that S is a perfect square?

A 1 B 2 C 4 D 8 E 64

24. The factorial of n, written $n!$, is defined by $n! = 1 \times 2 \times 3 \times \ldots \times (n-2) \times (n-1) \times n$. For how many positive integer values of k less than 50 is it impossible to find a value of n such that $n!$ ends in exactly k zeros?

A 0 B 5 C 8 D 9 E 10

2006…

5. Given that January 1st, 2006 fell on a Sunday, which day of the week will occur most frequently in 2007?

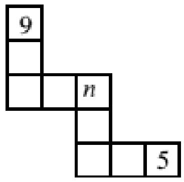
A Monday B Tuesday C Wednesday D Thursday E Friday

6. Which symbol should replace $\oplus$ to make the following equation true?

$$1 \times 2 \times (3 \oplus 4 + 5) \times (6 \times 7 + 8 + 9) = 2006.$$

A + B − C ÷ D × E none of these

10. The digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 are to be written in the squares so that every row and every column of three squares has a total of 13. Two numbers have already been entered. What is the value of n?



A 2 B 4 C 6 D 7 E 8

11. Three consecutive even numbers are such that the sum of four times the smallest and twice the largest exceeds three times the second by 2006. What is the sum of the digits of the smallest number?

A 8 B 11 C 14 D 17 E 20

14. Heather and Rachel each has some pennies. Heather has more than Rachel. In fact, the number of pennies that Heather has is the square of the number that Rachel has. The total number of pennies they have between them makes a whole number of pounds. What is the smallest this total could be?

A £1 B £6 C £57 D £99 E £101

19. An engineer is directed to a faulty signal, one quarter of the way into a tunnel. Whilst there, he is warned of a train heading towards the tunnel entrance. The engineer can run at 12 mph and can either run back to the tunnel entrance or forward to the exit. In either case, the engineer and the front of the train would reach the entrance or exit together. What is the speed in mph of the train?

A 16 B 20 C 24 D 32 E more information needed

25. X is a positive integer in which each digit is 1; that is, X is of the form 11111… .
Given that every digit of the integer $pX^2 + qX + r$ (where p, q and r are fixed integer coefficients and $p > 0$) is also 1, irrespective of the number of digits X, which of the following is a possible value of q?

A −2 B −1 C 0 D 1 E 2

2007…

2. This morning Sam told Pat "I am getting married today, aged 30." From this information, Pat may correctly deduce that Sam was born in:

A 1976 or 1977 B 1977 C 1978 D 1979 E 1977 or 1978

3. What is the value of $2006 \times 2008 - 2007 \times 2007$?

A -2007 B -1 C 0 D 1 E 4 026 042

6. Cheryl finds a bag of coins. There are 50 coins inside and the value of the contents is £1.81. Given that it contains only two-pence and five-pence coins, how many more five-pence coins are there inside the bag than two-pence coins?

A 4 B 6 C 8 D 10 E 12

7. How many whole numbers between 1 and 2007 are divisible by 2 but not by 7?

A 857 B 858 C 859 D 860 E 861

12. How many two-digit numbers N have the property that the sum of N and the number formed by reversing the digits of N is a square?

A 2 B 5 C 6 D 7 E 8

13. Which of the following gives the exact number of seconds in the last six complete weeks of 2007?

A 9! B 10! C 11! D 12! E 13!

{Note that $n! = 1 \times 2 \times 3 \times \ldots \times n$.*}*

18. The year 1789 (when the French Revolution started) has three and no more than three adjacent digits (7, 8 and 9) which are consecutive integers in increasing order. How many years between 1000 and 9999 have this property?

A 130 B 142 C 151 D 169 E 180

2008…

1. What is the value of $2 \times 2008 + 2008 \times 8$?

A 4016 B 16064 C 20080 D 64256 E 80020

2. A giant thresher shark weighing 1250 pounds, believed to be the heaviest ever caught, was landed by fisherman Roger Nowell off the Cornish coast in November 2007. The fish was sold by auction at Newlyn Fish Market for £255. Roughly, what was the cost per pound?

A 5p B 20p C 50p D £2 E £5

4. In this subtraction, P, Q, R and S are digits. What is the value of $P + Q + R + S$?

$$\begin{array}{r} 8\ Q\ 0\ S \\ -\ P\ 0\ R\ 2 \\ \hline 2\ 0\ 0\ 8 \end{array}$$

A 12 B 14 C 16 D 18 E 20

7. A newspaper headline read 'Welsh tortoise recaptured 1.8 miles from home after 8 months on the run'. Assuming the tortoise travelled in a straight line, roughly how many minutes did the tortoise take on average to 'run' one foot?
[1 mile = 5280 feet]

A 3 B 9 C 16 D 36 E 60

9. What is the remainder when the 2008-digit number 222 ... 22 is divided by 9?

A 8 B 6 C 4 D 2 E 0

12. Mr and Mrs Stevens were married on a Saturday in July 1948. On what day of the week did their diamond wedding anniversary fall this year?

A Monday B Tuesday C Thursday D Friday E Saturday

15. A sequence of positive integers $t_1, t_2, t_3, t_4, \ldots$ is defined by:
$t_1 = 13$; $t_{n+1} = \frac{1}{2}t_n$ if t_n is even; $t_{n+1} = 3t_n + 1$ if t_n is odd.
What is the value of t_{2008}?

A 1 B 2 C 4 D 8 E None of these.

2009…

2. Which of the following is not a multiple of 15?

A 135 B 315 C 555 D 785 E 915

4. Steve travelled 150 miles on a motorbike and used 10 litres of petrol. Given that 1 gallon ≈ 4.5 litres, roughly how many miles per gallon did Steve achieve on his journey?

A 10 B 20 C 40 D 50 E 70

10. Consider all three-digit numbers formed by using *different* digits from 0, 1, 2, 3 and 5. How many of these numbers are divisible by 6?

A 4 B 7 C 10 D 15 E 20

25. Four positive integers a, b, c and d are such that

$$abcd + abc + bcd + cda + dab + ab + bc + cd + da + ac + bd + a + b + c + d = 2009.$$

What is the value of $a + b + c + d$?

A 73 B 75 C 77 D 79 E 81

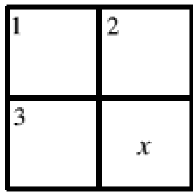
2010…

1. What is the digit x in this cross-number?

   Across
   1. A cube
   3. A cube

   Down
   1. One less than a cube

   A 2 B 3 C 4 D 5 E 6



2. What is the smallest possible value of $20p + 10q + r$ when p, q and r are *different* positive integers?

   A 31 B 43 C 53 D 63 E 2010

4. The year 2010 is one in which the sum of its digits is a factor of the year itself. How many more years will it be before this is next the case?

   A 3 B 6 C 9 D 12 E 15

13. How many two-digit numbers have remainder 1 when divided by 3 and remainder 2 when divided by 4?

    A 8 B 7 C 6 D 5 E 4

2011…

3. The robot *Lumber9* moves along the number line. *Lumber9* starts at 0, takes 1 step forward (to 1), then 2 steps backward (to -1), then 3 steps forward, 4 steps backward, and so on, moving alternately forwards and backwards, one more step each time. At what number is *Lumber9* after 2011 steps?

   A 1006 B 27 C 11 D 0 E -18

2012…

3. The price of my favourite soft drink has gone up by leaps and bounds over the past ten years. In four of those years it has leapt up by 5p each year, whilst in the other six years it has bounded up by 2p each year. The drink cost 70p in 2002. How much does it cost now?

   A £0.77 B £0.90 C £0.92 D £1.02 E £1.05

5. All six digits of three 2-digit numbers are different. What is the largest possible sum of three such numbers?

   A 237 B 246 C 255 D 264 E 273

6. What is the sum of the digits of the largest 4-digit palindromic number which is divisible by 15? [Palindromic numbers read the same backwards and forwards, e.g. 7227.]

A 18 B 20 C 24 D 30 E 36

10. Let N be the smallest positive integer whose digits add up to 2012. What is the first digit of $N + 1$?

A 2 B 3 C 4 D 5 E 6

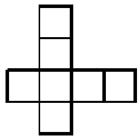
12. The number 3 can be expressed as the sum of one or more positive integers in four different ways:

$$3; \quad 1 + 2; \quad 2 + 1; \quad 1 + 1 + 1.$$

In how many ways can the number 5 be so expressed?

A 8 B 10 C 12 D 14 E 16

19. The numbers 2, 3, 4, 5, 6, 7, 8 are to be placed, one per square, in the diagram shown so that the sum of the four numbers in the horizontal row equals 21 and the sum of the four numbers in the vertical column also equals 21. In how many different ways can this be done?



A 0 B 2 C 36 D 48 E 72

2013…

1. Which of these is the largest number?

A $2 + 0 + 1 + 3$ B $2 \times 0 + 1 + 3$ C $2 + 0 \times 1 + 3$ D $2 + 0 + 1 \times 3$ E $2 \times 0 \times 1 \times 3$

2. Little John claims he is 2m 8cm and 3mm tall. What is this height in metres?

A 2.83m B 2.803m C 2.083m D 2.0803m E 2.0083m

7. In a 'ninety nine' shop, all items cost a number of pounds and 99 pence. Susanna spent £65.76. How many items did she buy?

A 23 B 24 C 65 D 66 E 76

13. Two entrants in a school's sponsored run adopt different tactics. Angus walks for half the time and runs for the other half, whilst Bruce walks for half the distance and runs for the other half. Both competitors walk at 3mph and run at 6mph. Angus takes 40 minutes to complete the course. How many minutes does Bruce take?

A 30 B 35 C 40 D 45 E 50

18. The numbers 2, 3, 12, 14, 15, 20, 21 may be divided into two sets so that the product of the numbers in each set is the same. What is this product?

A 420 B 1260 C 2520 D 6720 E 6350400

22. Consider numbers of the form $10n + 1$, where n is a positive integer. We shall call such a number 'grime' if it cannot be expressed as the product of two smaller numbers, possibly equal, both of which are of the form $10k + 1$, where k is a positive integer.
How many 'grime numbers' are there in the sequence 11, 21, 31, 41, ..., 981, 991?

A 0 B 8 C 87 D 92 E 99

2014…

1. What is 98×102?

A 200 B 9016 C 9996 D 998 E 99 996

3. December 31st 1997 was a Wednesday. How many Wednesdays were there in 1997?

A 12 B 51 C 52 D 53 E 365

5. How many integers between 1 and 2014 are multiples of both 20 and 14?

A 7 B 10 C 14 D 20 E 28

6. In the addition sum shown, each of the letters T, H, I and S represents a non-zero digit.
What is $T + H + I + S$?

$$\begin{array}{cccc} T & H & I & S \\ + & & I & S \\ \hline 2 & 0 & 1 & 4 \\ \hline \end{array}$$

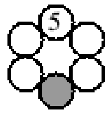
A 34 B 22 C 15 D 9 E
7

7. According to recent research, global sea levels could rise 36.8 cm by the year 2100 as a result of melting ice. Roughly how many millimetres is that per year?

A 10 B 4 C 1 D 0.4 E 0.1

2015…

6. The numbers 5, 6, 7, 8, 9, 10 are to be placed, one in each of the circles in the diagram, so that the sum of the numbers in each pair of touching circles is a prime number. The number 5 is placed in the top circle.
Which number is placed in the shaded circle?



A 6 B 7 C 8 D 9 E 10

10. The positive integer n is between 1 and 20. Milly adds up all the integers from 1 to n inclusive. Billy adds up all the integers from $n + 1$ to 20 inclusive. Their totals are the same.
What is the value of n?

A 11 B 12 C 13 D 14 E 15

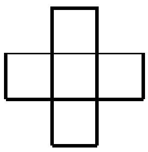
18. What is the largest integer k whose square k^2 is a factor of 10!?
[$10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.]

A 6 B 256 C 360 D 720 E 5040

2016…

1. How many times does the digit 9 appear in the answer to 987654321×9 ?

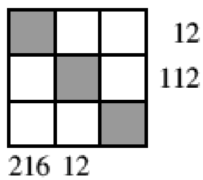
A 0 B 1 C 5 D 8 E 9

5. 

All the digits 2, 3, 4, 5 and 6 are placed in the grid, one in each cell, to form two three-digit numbers that are squares.
Which digit is placed in the centre of the grid?

A 2 B 3 C 4 D 5 E 6

10. The digits from 1 to 9 are to be written in the nine cells of the 3×3 grid shown, one digit in each cell.
The product of the three digits in the first row is 12.
The product of the three digits in the second row is 112.
The product of the three digits in the first column is 216.
The product of the three digits in the second column is 12.
What is the product of the digits in the shaded cells?



A 24 B 30 C 36 D 48 E 140

25. Let n be the smallest integer for which $7n$ has 2016 digits.
What is the units digit of n?

A 0 B 1 C 4 D 6 E 8

UKMT Numberwork Answers

2005…

1. **C** 2005 plus 2005 thousandths = 2005 + 2.005 = 2007.005.

5. **E** Four lengths of the indoor pool are equivalent to five lengths of the outdoor pool. So Rachel swam four ninths of the 63 days, that is 28 days, in the indoor pool.

7. **E** The sequences have common differences of 7 and 9 respectively. The lowest common multiple of 7 and 9 is 63, so the next term after 2005 to appear in both sequences is 2005 + 63, that is 2068.

12. **D** As $\sqrt{x}$ lies between 15 and 16, x lies between 225 and 256. The multiples of 7 in this interval are 231, 238, 245 and 252.

14. **C** When divided by 6, a whole number leaves remainder 0, 1, 2, 3, 4 or 5. So the possible remainders when a square number is divided by 6 are the remainders when 0, 1, 4, 9, 16 and 25 are divided by 6. These are 0, 1, 4, 3, 4 and 1 respectively, so a square number cannot leave remainder 2 (or remainder 5) when divided by 6.

22. **C** There are 81 terms in the series, so, using the formula $S = \frac{1}{2}n(a + l)$ for an arithmetic series:

$$S = \frac{81}{2}(x + 20 + x + 100) = 81(x + 60).$$

Now 81 is a perfect square, so S is a perfect square if and only if $x + 60$ is a perfect square. As x is a positive integer, the smallest possible value of x is 4.

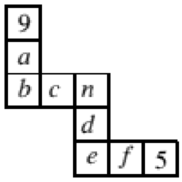
24. **D** When $n!$ is written in full, the number of zeros at the end of the number is equal to the power of 5 when $n!$ is written as the product of prime factors, because there is at least that high a power of 2 available. For example, $12! = 1 \times 2 \times 3 \times \ldots \times 12 = 2^{10} \times 3^5 \times 5^2 \times 7 \times 11$.
This may be written as $2^8 \times 3^5 \times 7 \times 11 \times 10^2$, so 12! ends in 2 zeros, as $2^8 \times 3^5 \times 7 \times 11$ is not a multiple of 10.
We see that 24! ends in 4 zeros as 5, 10, 15 and 20 all contribute one 5 when 24! is written as the product of prime factors, but 25! ends in 6 zeros because $25 = 5 \times 5$ and hence contributes two 5s. So there is no value of n for which $n!$ ends in 5 zeros.
Similarly, there is no value of n for which $n!$ ends in 11 zeros since 49! ends in 10 zeros and 50! ends in 12 zeros. The full set of values of k less than 50 for which it is impossible to find a value of n such that $n!$ ends in k zeros is 5, 11, 17, 23, 29, 30 (since 124! ends in 28 zeros and 125! ends in 31 zeros), 36, 42, 48.

2006…

5. **A** As 2006 is not a leap year, January 1st, 2007 will fall one day later in the week than January 1st, 2006, that is on a Monday. So there will be 53 Mondays in 2007 and 52 of each of the other days of the week.

6. **D** $1 \times 2 \times (3 \oplus 4 + 5) \times (6 \times 7 + 8 + 9) = 2006$, that is $2 \times (3 \oplus 4 + 5) \times (42 + 8 + 9) = 2006$, that is $(3 \oplus 4 + 5) \times 59 = 1003$, that is $3 \oplus 4 + 5 = 17$, that is $3 \oplus 4 = 12$. So $\oplus$ should be replaced by $\times$.

10. **B** Let a, b, c, d, e, f be the numbers in the squares shown. Then the sum of the numbers in the four lines is $1 + 2 + 3 + \ldots + 9 + b + n + e$ since each of the numbers in the corner squares appears in exactly one row and one column. So $45 + b + n + e = 4 \times 13 = 52$, that is $b + n + e = 7$. Hence b, n, e are 1, 2, 4 in some order.



If $b = 2$ then $a = 2$; if $b = 4$ then $a = 0$. Both cases are impossible, so $b = 1$ and $a = 3$.
This means that $n = 2$ or $n = 4$. However, if $n = 2$ then $c = 10$, so $n = 4$ and $c = 8$.
(*The values of the other letters are* $e = 2, d = 7, f = 6$.)

11. **E** Let the smallest of the three even numbers be n. Then the other two numbers are $n + 2$ and $n + 4$. So $4n + 2(n + 4) = 3(n + 2) + 2006$, that is $6n + 8 = 3n + 2012$, that is $n = 668$.

14. **B** Let Rachel and Heather have x and x^2 pennies respectively. So $x + x^2 = 100n$, where x and n are positive integers. We require, therefore, that $x(x + 1) = 100n = 2^2 \times 5^2 \times n$. Now x and $x + 1$ cannot both be multiples of 5, so their product will be a multiple of 25 if and only if x or $x + 1$ is a multiple of 25. The smallest value of x which satisfies this condition is 24 which is a multiple of 4 so 24×25 is a multiple of 100. Therefore Rachel has 24 pennies, Heather has 576 pennies and, in total, they have £6.

19. **C** Let the length of the tunnel and the distance from the front of the train to the entrance of the tunnel when the engineer receives the warning be l and x respectively. If the engineer runs to the exit of the tunnel, he will take three times as long as he would if he ran to the entrance. So the train takes three times as long to travel a distance $x + l$ as it does to travel a distance x. Hence $l = 2x$. The train, therefore, travels a distance x in the same time that the engineer would take to travel $\frac{1}{4}l$, that is to travel $\frac{1}{2}x$. So the speed of the train is twice that of the engineer.

25. **E** Let X consist of x digits, each of which is 1. So $X = \frac{10^x - 1}{9}$. Let $pX^2 + qX + r$ consist of y digits, each of which is 1. So $pX^2 + qX + r = \frac{10^y - 1}{9}$. Then $p\left(\frac{10^x-1}{9}\right)^2 + q\left(\frac{10^x-1}{9}\right) + r = \frac{10^y-1}{9}$, that is $p(10^{2x} - 2 \times 10^x + 1) + 9q(10^x - 1) + 81r = 9(10^y - 1)$, that is (on dividing throughout by 10^{2x}) $p + (9q - 2p)10^{-x} + (p - 9q + 81r)10^{-2x} = 9 \times 10^{y-2x} - 9 \times 10^{-2x}$.
We now let x tend to infinity (through integer values). The LHS of the above equation tends to p, and the second term on the right goes to 0. By continuity of the function $f(u) = 10^u = e^{u \log 10}$, we can deduce that $y - 2x$ must tend to a limit. Let this limit be L. Since $y - 2x$ is always an integer, it must actually equal L for all x sufficiently large. Passing to the limit, therefore, we obtain $p = 9 \times 10^L$. Since p is to be an integer, we must have that L (also an integer) is a non-negative integer. Substituting for p in the previous equation and simplifying leads to

$$9q - 18 \times 10^L + (9 \times 10^L - 9q + 81r)10^{-x} = -9 \times 10^{-x}.$$

Passing to the limit again leads to $q = 2 \times 10^L$ and the previous line then also gives $9 \times 10^L - 18 \times 10^L + 81r = -9$. So $r = \frac{10^L - 1}{9}$.
Possible values of (p, q, r) therefore are $(9, 2, 0)$, $(90, 20, 1)$, $(900, 200, 11)$, etc. So of the values given in the question for q, only $q = 2$ is possible.
(Observe that the three triples above correspond to $L = 0, L = 1, L = 2$ respectively and we note that increasing L by 1 corresponds to multiplying $pX^2 + qX + r$ by 10 and adding 1. As $pX^2 + qX + r$ consists only of 1s, $10(pX^2 + qX + r) + 1$ will also consist only of 1s, explaining why there is an infinite family of quadratics which satisfy the required condition.)

2007…

2. **A** If Sam's birthday falls before 9 November, then the fact that she is aged 30 on 8 November means that she was born in 1977. However, if her birthday falls on 9 November or later then her 31st birthday will fall in 2007, which means that she was born in 1976.

3. **B** In general, $(n - 1) \times (n + 1) - n^2 = n^2 - 1 - n^2 = -1$. This applies with $n = 2007$.

6. **A** Let the number of five-pence coins be x. Then $5x + 2(50 - x) = 181$, that is $3x = 81$, that is $x = 27$. So there are 27 five-pence coins and 23 two-pence coins.

7. **D** There are 1003 whole numbers between 1 and 2007 which are divisible by 2. Those which are also divisible by 7 are the multiples of 14, namely 14, 28, 42, …, 2002. There are 143 of these, so the required number is $1003 - 143 = 860$.

12. **E** Let N be the two-digit number 'ab', that is $N = 10a + b$. So the sum of N and its 'reverse' is $10a + b + 10b + a = 11a + 11b = 11(a + b)$. As 11 is prime and a and b are both single digits, $11(a + b)$ is a square if, and only if, $a + b = 11$. So the possible values of N are 29, 38, 47, 56, 65, 74, 83, 92.

13. **B** The exact number of seconds in six complete weeks is $6 \times 7 \times 24 \times 60 \times 60 = 6 \times 7 \times (3 \times 8) \times (2 \times 5 \times 6) \times (3 \times 4 \times 5) = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = 10!$.

18. **A** There are 9 years of the form $123n$ as n may be any digit other than 4. Similarly, there are 9 years each of the forms $234n$, $345n$, $456n$, $567n$ and $678n$, but 10 years of the form $789n$ as, in this case, n may be *any* digit. There are also 9 years of the form $n012$ and 9 of the form $n123$, as in both cases n may be any digit other than 0. However, there are 8 years of the form $n234$ as in this case n cannot be 0 or 1. Similarly, there are 8 years each of the forms $n345$, $n456$, $n567$, $n678$ and $n789$.
So the total numbers of years is $1 \times 10 + 8 \times 9 + 6 \times 8 = 130$.

2008…

1. **C** $2 \times 2008 + 2008 \times 8 = 10 \times 2008 = 20080$.

2. **B** The cost per pound is $£\frac{255}{1250} \approx £\frac{1}{5} = 20\text{ p}$.

4. **C** From the units column we see that $S = 0$. Then the tens column shows that $R = 9$, the hundreds column that $Q = 1$, and the thousands that $P = 6$. So $P + Q + R + S = 16$.

7. **D** In 1.8 miles there are 1.8×5280 feet $= 18 \times 528$ feet, while in 8 months there are roughly $8 \times 30 \times 24 \times 60$ minutes. Hence the time to 'run' one foot in minutes is roughly $\frac{10 \times 30 \times 20 \times 60}{20 \times 500} = 36$ minutes.

9. **D** A number is divisible by 9 if, and only if, the sum of its digits is divisible by 9. The given number is $N + 2$, where $N = 222\ldots220$ has 2007 2s. Since $2007 = 223 \times 9$, N is divisible by 9 and the required remainder is therefore 2.

12. **C** There are 365 days in a normal year and 366 in a leap year. Apart from certain exceptions (none of which occurs in this period) a leap year occurs every 4 years. Now $365 = 7 \times 52 + 1$ and $366 = 7 \times 52 + 2$. Hence each date moves on by 5 days every 4 years. So in 60 years, it moves on 75 days. Since $75 = 7 \times 10 + 5$, that means it moves on to a Thursday.

15. **A** The sequence proceeds as follows: 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1 … . The block 4, 2, 1 repeats *ad infinitum* starting after t_7. But $2008 - 7 = 2001$ and $2001 = 3 \times 667$. Hence t_{2008} is the third term in the 667th such block and is therefore 1.

2009…

2. **D** $\frac{785}{15} = 52\frac{1}{3}$ hence 785 is not a multiple of 15. But $\frac{135}{15} = 9$, $\frac{315}{15} = 21$, $\frac{555}{15} = 37$, $\frac{915}{15} = 61$.

4. **E** Steve achieved $\frac{150}{10} \times 4.5$ miles per gallon which is $15 \times 4.5 = 67.5 \approx 70$.

10. **B** A number is a multiple of 6 precisely when it is both a multiple of 2 and of 3. To be a multiple of 2, it will need to end with an even digit; i.e. 0 or 2. If it ends with 0, the sum of the other two digits must be a multiple of 3; and only $3 = 1 + 2$ or $6 = 1 + 5$ are possible. That gives the numbers 120, 210, 150, 510. If it ends with 2, the sum of the others must be $1 = 0 + 1$ or $4 = 1 + 3$. That gives 102, 132 and 312.

25. **A** The left-hand side of the equation can be written as

$$(a + 1)(b + 1)(c + 1)(d + 1) - 1.$$

Hence

$$(a + 1)(b + 1)(c + 1)(d + 1) = 2010.$$

Now expressing 2010 as a product of primes gives $2010 = 2 \times 3 \times 5 \times 67$ hence $a + b + c + d = 1 + 2 + 4 + 66 = 73$.

2010…

1. **C** The only two-digit cubes are 27 and 64. As 1 *Down* is one less than a cube then 3 *Across* must start with 6 or 3 and so is 64. Thus $x = 4$.

2. **B** The smallest possible value is attained by using $p = 1$, $q = 2$ and $r = 3$. Therefore this value is $20 \times 1 + 10 \times 2 + 3 = 43$.

4. **B** $2 + 0 + 1 + 1 = 4$. Multiples of 4 are even, hence 2011 is not valid and the same argument applies to 2013, 2015, 2017 and 2019.
$2 + 0 + 1 + 2 = 5$. The units digit for multiples of 5 is 0 or 5, hence 2012 is not valid.
$2 + 0 + 1 + 4 = 7$. But $\frac{2014}{7} = 287\frac{5}{7}$, hence 2014 is not valid.
$2 + 0 + 1 + 6 = 9$. Since $2016 = 9 \times 224$, 2016 is valid.
Hence we have to wait $2016 - 2010 (= 6)$ more years.

13. **A** The lowest common multiple of 3 and 4 is 12. Hence both of the required conditions are satisfied only by numbers that are 2 less than multiples of 12 and also less than 100, ie: 10, 22, 34, 46, 58, 70, 82 and 94.
Therefore 8 two-digit numbers satisfy the conditions.

2011…

3. A After the n th step, *Lumber9* is at number: $\begin{cases} \frac{n+1}{2} & \text{for } n \text{ odd,} \\ \frac{-n}{2} & \text{for } n \text{ even.} \end{cases}$

Hence when $n = 2011$, *Lumber9* is at number $\frac{2011+1}{2} = 1006$.

2012…

3. D The cost now is $(70 + 4 \times 5 + 6 \times 2)\text{p} = £1.02$.

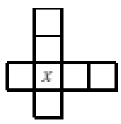
5. C Let the required addition be 'ab' + 'cd' + 'ef', where a, b, c, d, e, f are single, distinct digits. To make this sum as large as possible, we need a, c, e (the tens digits) as large as possible; so they must be 7, 8, 9 in some order. Then we need b, d, f as large as possible, so 4, 5, 6 in some order. Hence the largest sum is $10(7 + 8 + 9) + (4 + 5 + 6) = 10 \times 24 + 15 = 255$.

6. C In order to be a multiple of 15, a number must be a multiple both of 3 and of 5. So its units digit must be 0 or 5. However, the units digit must also equal the thousands digit and this cannot be 0, so the required number is of the form '5aa5'. The largest such four-digit numbers are 5995, 5885, 5775. Their digit sums are 28, 26, 24 respectively. In order to be a multiple of 3, the digit sum of a number must also be a multiple of 3, so 5775 is the required number. The sum of its digits is 24.

10. E It can be deduced that N must consist of at least 224 digits since the largest 223-digit positive integer consists of 223 nines and has a digit sum of 2007. It is possible to find 224-digit positive integers which have a digit sum of 2012.
The largest of these is 99 999 ...999 995 and the smallest is 59 999 ...999 999.
So $N = 59\,999\ldots\,999\,999$ and $N + 1 = 60\,000\ldots\,000\,000$ (223 zeros).

12. E Two different ways of expressing 5 are 1 + 4 and 4 + 1. In the following list these are denoted as {1, 4: two ways}. The list of all possible ways is {5: one way}, {2, 3: two ways}, {1, 4: two ways}, {1, 2, 2: three ways}, {1, 1, 3: three ways}, {1, 1, 1, 2: four ways}, {1, 1, 1, 1, 1: one way}. So in total there are 16 ways.
{*Different expressions of a positive integer in the above form are known as 'partitions'. It may be shown that the number of distinct compositions of a positive integer n is* 2^{n-1}.}

19. E Note that the number represented by x appears in both the horizontal row and the vertical column. Note also that $2 + 3 + 4 + 5 + 6 + 7 + 8 = 35$. Since the numbers in the row and those in the column have sum 21, we deduce that $x = 2 \times 21 - 35 = 7$.



We now need two disjoint sets of three numbers chosen from 2, 3, 4, 5, 6, 8 so that the numbers in both sets total 14. The only possibilities are {2, 4, 8} and {3, 5, 6}. We have six choices of which number to put in the top space in the vertical line, then two for the next space down and one for the bottom space. That leaves three choices for the first space in the horizontal line, two for the next space and one for the final space. So the total number of ways is $6 \times 2 \times 1 \times 3 \times 2 \times 1 = 72$.

2013…

1. **A** Calculating the value of each option gives $2 + 0 + 1 + 3 = 6$, $2 \times 0 + 1 + 3 = 4$, $2 + 0 \times 1 + 3 = 5$, $2 + 0 + 1 \times 3 = 5$ and $2 \times 0 \times 1 \times 3 = 0$ so $2 + 0 + 1 + 3$ is the largest.

2. **C** In metres, the height 2m 8cm and 3mm is $2 + 8 \times 0.01 + 3 \times 0.001 = 2 + 0.08 + 0.003 = 2.083$m.

7. **B** The first item that Susanna buys makes her bill a number of pounds and 99 pence. Each extra item she buys after that decreases by one the number of pence in her total bill. Let n be the number of items bought. To be charged £65.76, $1 + 99 - n = 76$ so $n = 100 - 76 = 24$. Alternatives of 124 items or more are infeasible as they would each give a total greater than £65.76.

13. **D** Angus completes the course in 40 minutes, so he spends 20 minutes (which is $\frac{1}{3}$ of an hour) walking and the same time running. By using distance = speed × time, the length of the course is $3 \times \frac{1}{3} + 6 \times \frac{1}{3} = 1 + 2 = 3$ miles.
Bruce completes the course by walking for $1\frac{1}{2}$ miles and running for $1\frac{1}{2}$ miles. So, by using time $= \dfrac{\text{distance}}{\text{speed}}$, Bruce's total time in hours is $\dfrac{1\frac{1}{2}}{3} + \dfrac{1\frac{1}{2}}{6} = \dfrac{1}{2} + \dfrac{1}{4} = \dfrac{3}{4}$ of an hour. So Bruce takes 45 minutes to complete the course.

18. **C** The product of all the numbers in the list is $2 \times 3 \times 12 \times 14 \times 15 \times 20 \times 21$ which, when expressed in terms of prime factors is $2 \times 3 \times 2 \times 2 \times 3 \times 2 \times 7 \times 3 \times 5 \times 2 \times 2 \times 5 \times 3 \times 7$ which is equal to $2^6 \times 3^4 \times 5^2 \times 7^2 = (2^3 \times 3^2 \times 5 \times 7)^2 = 2520^2$. The answer 2520 is expressible as both $2 \times 3 \times 20 \times 21$ and $12 \times 14 \times 15$.

22. **C** The numbers in the sequence 11, 21, 31, 41, …. , 981, 991 are of the form $10n + 1$ for $n = 1$ to 99. There are therefore 99 numbers in this sequence.
Twelve terms of this sequence can be expressed using factors of the form $10k + 1$. In this form, these terms are 11×11, 11×21, 11×31, …, 11×81 and 21×21, 21×31, 21×41 and 31×31. All other pairings give products that are too large.
Hence, there are $99 - 12 = 87$ 'grime' numbers.

2014…

1. **C** $98 \times 102 = (100 - 2)(100 + 2) = 10000 - 4 = 9996$.

3. **D** The year 1997 was not a leap year so had $365 = 52 \times 7 + 1$ days. Hence, starting from 1st January, 1997 had 52 complete weeks, each starting with the same day as 1st January, followed by 31st December. As 31st December was a Wednesday, so too were all the first days of the 52 complete weeks. So there were 53 Wednesdays in 1997.

5. **C** The prime factorisations of 20 and 14 are $20 = 2 \times 2 \times 5$ and $14 = 2 \times 7$. The lowest common multiple of 20 and 14 is 140 as $140 = 2 \times 2 \times 5 \times 7$. For a number to be a multiple of 20 and 14 it must be a multiple of 140. As $2014 \div 140 = 14$ remainder 54, there are 14 integers in the required range. Note: The integer 0, which is also a multiple of 20 and of 14 is excluded as we are considering numbers *between* 1 and 2014.

6. **B** Working from right to left, the units column shows that $S = 2$ or 7. If $S = 2$, then $I + I = 1$ or 11, neither of which is possible. Hence $S = 7$ and it follows that $I + I = 0$ or 10. However, as the digits are non-zero, $I = 5$. The hundreds column then shows that $H = 9$ and so $T = 1$. This gives $T + H + I + S = 1 + 9 + 5 + 7 = 22$.

7. **B** Since $36.8 \div 86$ is approximately $40 \div 100 = 0.4$, the sea level rises by roughly 0.4 cm, which is 4 mm, per year.

2015…

6. **E** The prime numbers which are the sums of pairs of numbers in touching circles are all odd as they are greater than 2. This means that any two adjacent circles in the diagram must be filled with one odd number and one even number. The number 10 may not be placed on either side of 5, since $10 + 5 = 15 = 3 \times 5$. So either side of the 5 must be 6 and 8. Below 6 and 8 must be 7 and 9 respectively leaving 10 to be placed in the shaded circle at the bottom.

10. **D** The total of the numbers from 1 to 20 is $\frac{1}{2} \times 20 \times (20 + 1) = 210$. If Milly and Billy have totals which are equal, their totals must each be 105. Milly's total, of the numbers from 1 to n, is $\frac{1}{2}n(n+1)$ so $\frac{1}{2}n(n+1) = 105$ which gives $n^2 + n = 210$. Therefore $n^2 + n - 210 = 0$ which factorises to give $(n + 15)(n - 14) = 0$. As n is a positive integer, $n = 14$.

18. **D** Expressed as a product of its prime factors, 10! is $2 \times 5 \times 3 \times 3 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3 \times 5 \times 2 \times 2 \times 3 \times 2$ which is $2^8 \times 3^4 \times 5^2 \times 7$. This can be written as $(2^4 \times 3^2 \times 5)^2 \times 7$ so the largest integer k such that k^2 is a factor of 10! is $2^4 \times 3^2 \times 5$ which is 720.

2016…

1. **B** Since the answer to 987654321 × 9 is 8 888 888 889, the digit 9 appears once.

5. **A** From the available digits the squares could be 256, 324 or 625. Since the middle digits must be the same, the centre digit must be 2.

10. **B** None of the products for the first two rows and first two columns contains a factor of 5, so the bottom right cell must contain the 5. The prime factorisation of 112 is $2^4 \times 7$ and, as 7 is not a factor of 216 or 12, then 7 must be in the right cell of the middle row. The remaining 2^4 must be the product of two different numbers, namely 8 and 2. The 2 must be in the centre cell as 8 is not a factor of 12. The grid is now as shown above. The prime factorisation of 216 is $2^3 \times 3^3$ and the 3^3 must be the product of a 3 and a 9.

8	2	7
		5

The 3 must be in the top left cell as the product of the top row is 12 which is not a multiple of 9. Thus, the product of the three shaded cells is 3 × 2 × 5 which is 30. The completed grid is as shown on the right.

3	1	4	12
8	2	7	112
9	6	5	
216	12		

25. **D** For n to be the smallest integer for which $7n$ has 2016 digits, $7n$ must start with 1, be followed by 2014 zeros and end with a digit a. When this number is divided by 7, the answer is formed from the repeating sequence of 6 digits 142857. The remainders also form a repeating sequence 3, 2, 6, 4, 5, 1. These sequences are repeated 335 times as 6 × 335 is 2010. The last 4 zeros (to make 2014 zeros in total) and the final a create the last section of the division as shown:

$$\begin{array}{r} \ldots\ldots\ldots 1\ 4\ 2\ 8 \\ \hline \ldots\ {}^{1}0^{3}0^{2}0^{6}0^{4}a \end{array}.$$

Finally, $40 + a$ must be divisible by 7 and be as small as possible. So $a = 2$ and as $42 \div 7 = 6$ the units digit of n is 6.