UKMT Ratio Questions

(Answers follow after all the questions)

2005…

10. Sam and Pat were counting their money. They discovered that if Sam gave Pat £5, then Pat would have 5 times as much as Sam, but if Pat gave Sam £5, then Sam would have 5 times as much as Pat. How much did they have altogether?

A £10 B £15 C £20 D £25 E £30

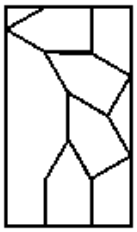
2008…

17. Andy and his younger cousin Alice both have their birthdays today. Remarkably, Andy is now the same age as the sum of the digits of the year of his birth and the same is true of Alice. How many years older than Alice is Andy?

A 10 B 12 C 14 D 16 E 18

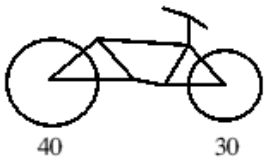
22. A pentagon is made by attaching an equilateral triangle to a square with the same edge length. Four such pentagons are placed inside a rectangle, as shown.
What is the ratio of the length of the rectangle to its width?

A $\sqrt{3}:1$ B $2:1$ C $\sqrt{2}:1$ D $3:2$ E $4:\sqrt{3}$



2009…

5. Boris Biker entered the Tour de Transylvania with an unusual bicycle whose back wheel is larger than the front. The radius of the back wheel is 40 cm, and the radius of the front wheel is 30 cm. On the first stage of the race the smaller wheel made 120000 revolutions. How many revolutions did the larger wheel make?



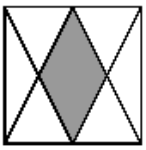
A 90000 B 90000π C 160000 D $\frac{160000}{\pi}$ E 120000

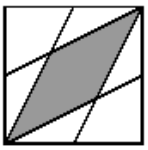
19. Hamish and his friend Ben live in villages which are 51 miles apart. During the summer holidays, they agreed to cycle towards each other along the same main road. Starting at noon, Hamish cycled at x mph. Starting at 2 pm, Ben cycled at y mph. They met at 4 pm. If they had both started at noon, they would have met at 2.50 pm. What is the value of y?

A 7.5 B 8 C 10.5 D 12 E 12.75

2010...

19. The diagrams show two different shaded rhombuses, each inside a square with sides of length 6.
Each rhombus is formed by joining vertices of the square to midpoints of the sides of the square. What is the difference between the shaded areas?

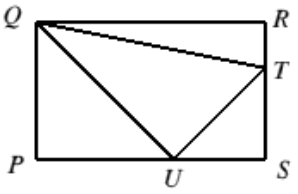




A 4 B 3 C 2 D 1 E 0

2011...

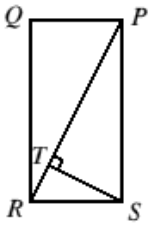
16. $PQRS$ is a rectangle. The area of triangle QRT is $\frac{1}{5}$ of the area of $PQRS$, and the area of triangle TSU is $\frac{1}{8}$ of the area of $PQRS$. What fraction of the area of rectangle $PQRS$ is the area of triangle QTU?



A $\frac{27}{40}$ B $\frac{21}{40}$ C $\frac{1}{2}$ D $\frac{19}{40}$ E $\frac{23}{60}$

2013...

14. The diagram shows a rectangle $PQRS$ in which $PQ : QR = 1 : 2$. The point T on PR is such that ST is perpendicular to PR. What is the ratio of the area of the triangle RST to the area of the rectangle $PQRS$?



A $1 : 4\sqrt{2}$ B $1 : 6$ C $1 : 8$ D $1 : 10$ E $1 : 12$

20. The ratio of two positive numbers equals the ratio of their sum to their difference. What is this ratio?

A $(1+\sqrt{3}):2$ B $\sqrt{2} : 1$ C $(1+\sqrt{5}):2$ D $(2+\sqrt{2}):1$ E $(1+\sqrt{2}):1$

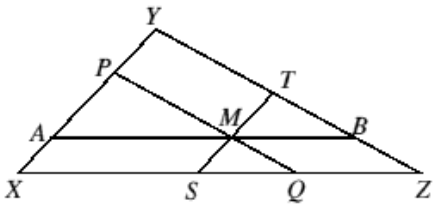
2014...

12. Karen has three times the number of cherries that Lionel has, and twice the number of cherries that Michael has. Michael has seven more cherries than Lionel. How many cherries do Karen, Lionel and Michael have altogether?

A 12 B 42 C 60 D 77 E 84

2015…

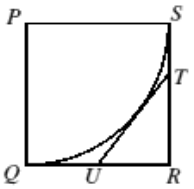
21. The diagram shows a triangle XYZ. The sides XY, YZ and XZ have lengths 2, 3 and 4 respectively. The lines AMB, PMQ and SMT are drawn parallel to the sides of triangle XYZ so that AP, QS and BT are of equal length. What is the length of AP?



A $\frac{10}{11}$ B $\frac{11}{12}$ C $\frac{12}{13}$ D $\frac{13}{14}$ E $\frac{14}{15}$

2016…

24. The diagram shows a square $PQRS$. The arc QS is a quarter circle. The point U is the midpoint of QR and the point T lies on SR. The line TU is a tangent to the arc QS. What is the ratio of the length of TR to the length of UR?



A 3 : 2 B 4 : 3 C 5 : 4 D 7 : 6 E 9 : 8

UKMT Ratio Answers

2005...

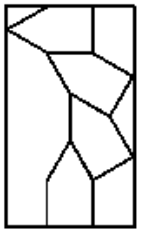
10. **B** Let Sam and Pat have £x and £y respectively.
Then $y + 5 = 5(x - 5)$, that is $y = 5x - 30$. Also, $x + 5 = 5(y - 5)$, that is $x = 5y - 30$. Solving these simultaneous equations gives $x = 7.5$ and $y = 7.5$ so the friends have £15 altogether.
(Note: from the information given, we may deduce that Sam and Pat have the same amount of money and this leads to a shorter method: $x + 5 = 5(x - 5)$, that is $x = 5x - 30$, that is $x = 7.5$.)

2008...

17. **E** Let 'X' be a single digit. If $2008 - 200X = 2 + 0 + 0 + X$ then $8 - X = 2 + X$ so $X = 3$. So Alice (being the younger) could have been born in 2003. Next if $2008 - 199X = 1 + 9 + 9 + X$ then $18 - X = 19 + X$, which is impossible. Similarly if $2008 - 198X = 1 + 9 + 8 + X$ then $28 - X = 18 + X$, so $X = 5$. Thus Alice or Andy could have been born in 1985. Finally if $2008 - 19YX = 1 + 9 + X + Y$ for some digit $Y \leqslant 7$, then $108 - YX = 10 + Y + X$. Hence $98 = YX + Y + X$ which is impossible, since $YX + Y + X$ is at most $79 + 7 + 9 = 95$. Hence there are no more possible dates and so Andy was born in 1985 and Alice in 2003.

22. **A** Let r be the length of a side of the equilateral triangle.
Hence the width of the rectangle is $r\sin 60° + r + r\sin 60° = r(1 + 2\sin 60°) = r(1 + \sqrt{3})$ and its length is $3r + 2r\sin 60° = r(3 + \sqrt{3})$.
So the ratio of the length to the width is

$$(3 + \sqrt{3}) : (1 + \sqrt{3}) = \sqrt{3}(1 + \sqrt{3}) : (1 + \sqrt{3}) = \sqrt{3} : 1.$$



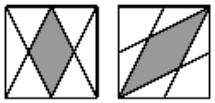
2009...

5. **A** As the ratio of the radii is 3 : 4 then the number of revolutions made by the larger wheel is $120000 \times \frac{3}{4} = 90000$.

19. **C** The distance cycled by Hamish between noon and 4 pm is $4x$.
The distance cycled by Ben between 2 pm and 4 pm is $2y$.
They meet at 4 pm hence $4x + 2y = 51$ or $2x + 2(x + y) = 51$ (*).
If they had both started at noon then they would have met at 2:50 pm and so $2\frac{5}{6}(x + y) = 51$.
Hence $x + y = 51 \times \dfrac{6}{17} = 18$. Hence from (*) $2x + 2 \times 18 = 51$.
Hence $2x = 15$ giving $x = 7\frac{1}{2}$. Thus $y = 10\frac{1}{2}$.

2010...

19. **B** In the rhombus on the left, drawing vertical straight lines at distances of $1\frac{1}{2}$, 3 and $4\frac{1}{2}$ from the left edge of the square, and a horizontal straight line bisecting the square, creates 16 equivalent triangles. Of these, four are shaded giving a total shaded area of $\frac{1}{4} \times 6 \times 6 = 9$.



Draw in the diagonal from NW to SE in the rhombus on the right. The four unshaded triangles now above the shaded area are all equal in area (a say); and one can see that 3 of these together make up $\frac{1}{4}$ of the square. Hence $a = 3$. Thus the shaded area equals $36 - 3 \times 8 = 12$.
Therefore the difference between the shaded areas is $12 - 9 = 3$.

2011...

16. **E** Let $QR = x$ and $RS = y$ in the rectangle $PQRS$. Hence the area of $PQRS$ is xy.
The area of triangle QRT is $\frac{1}{2}RT \times x = \frac{1}{5}xy$, hence $RT = \frac{2}{5}y$. Thus $TS = RS - RT = \frac{3}{5}y$.
The area of triangle TSU is $\frac{1}{2}SU \times \frac{3}{5}y = \frac{1}{8}xy$, hence $SU = \frac{5}{12}x$.
Therefore the area of triangle PUQ is $\frac{1}{2} \times \frac{7}{12}xy = \frac{7}{24}xy$.
Hence, as a fraction of the area of rectangle $PQRS$, the area of triangle QTU is

$$\frac{xy\left(1 - \frac{1}{5} - \frac{1}{8} - \frac{7}{24}\right)}{xy} = \frac{23}{60}.$$

2013...

14. **D** Triangle RST is similar to triangle RPS as their corresponding angles are equal. Using Pythagoras' Theorem, the ratio of RS to RP is $1 : \sqrt{5}$. So the ratio of RT to RS is also $1 : \sqrt{5}$. Therefore the ratio of the area of the triangle RST to the area of triangle RPS is $1 : 5$. Triangle RPS is half the rectangle $PQRS$, so the ratio of the area of triangle RST to the area of rectangle $PQRS$ is $1 : 10$.

20. **E** Let the two positive numbers be x and y with $x > y$. The sum of the numbers is greater than their difference, so the two ratios which are equal are $x : y$ and $x + y : x - y$. Therefore $\frac{x}{y} = \frac{x+y}{x-y}$. By dividing the top and bottom of the right-hand side by y we obtain $\frac{x}{y} = \frac{\frac{x}{y}+1}{\frac{x}{y}-1}$.
Letting $k = \frac{x}{y}$ gives $k = \frac{k+1}{k-1}$ which gives the quadratic $k^2 - 2k - 1 = 0$. Completing the square gives $(k-1)^2 = 2$ whence $k = 1 \pm \sqrt{2}$. However, as x and y are both positive, $k \neq 1 - \sqrt{2}$. As the ratio $\frac{x}{y} = 1 + \sqrt{2}$, the ratio $x : y$ is $1 + \sqrt{2} : 1$.

2014...

12. **D** Let Lionel have x cherries. Michael then has $(x + 7)$ cherries. Karen's number of cherries is described in two ways. She has $3x$ cherries and also $2(x + 7)$ cherries. So $3x = 2x + 14$ and therefore $x = 14$. Lionel has 14 cherries, Michael 21 cherries and Karen 42 cherries giving a total of $14 + 21 + 42 = 77$ cherries.

2015...

21. **C** All of the triangles in the diagram are similar as they contain the same angles. The sides of each triangle are therefore in the ratio 2 : 3 : 4. First consider triangle APM. Let $AP = x$, so that $AM = 2x$. Now considering triangle TBM, as $BT = x$, $BM = \frac{4x}{3}$. The quadrilateral $AMSX$ is a parallelogram as AM is parallel to XS and MS is parallel to AX. So $AM = XS = 2x$. Similarly $QZ = BM = \frac{4x}{3}$. Considering the base of triangle XYZ, $XS + SQ + QZ = 4$. So $2x + x + \frac{4x}{3} = 4$ and therefore $x = \frac{12}{13}$.

2016...

24. **B** Let the square have side-length 2, $RT = h$ and let A be the point of contact between TU and the circle. Two tangents to a circle which meet at a point are of equal length. So as $QU = 1$ so does AU. Similarly $TA = TS = 2 - h$. Applying Pythagoras' Theorem to triangle URT gives $1^2 + h^2 = (1 + 2 - h)^2$ so $1 + h^2 = 9 - 6h + h^2$ and therefore $8 - 6h = 0$ which gives $h = \frac{4}{3}$. The required ratio is then 4 : 3.