UKMT Trigonometry Questions

(Answers follow after all the questions)

2005...

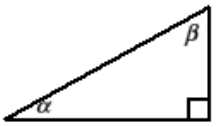
20. It takes two weeks to clean the 3312 panes of glass in the 6000m^2 glass roof of the British Museum, a task performed once every two years. Assuming that all the panes are equilateral triangles of the same size, roughly how long is the side of each pane?

A 50 cm B 1 m C 2 m D 3 m E 4 m

2006...

16. If $\alpha < \beta$, how many different values are there among the following expressions?

$\sin\alpha \sin\beta$ $\sin\alpha \cos\beta$ $\cos\alpha \sin\beta$ $\cos\alpha \cos\beta$

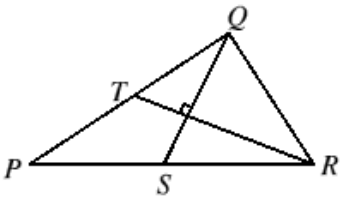


A 1 B 2 C 3 D 4 E It depends on the value of α

2007...

22. In triangle PQR, S and T are the midpoints of PR and PQ respectively; QS is perpendicular to RT; $QS = 8$; $RT = 12$.
What is the area of triangle PQR?

A 24 B 32 C 48 D 64 E 96



2008...

24. The length of the hypotenuse of a particular right-angled triangle is given by $\sqrt{1+3+5+7+\ \ldots\ +25}$. The lengths of the other two sides are given by $\sqrt{1+3+5+\ \ldots\ +x}$ and $\sqrt{1+3+5+\ \ldots\ +y}$ where x and y are positive integers.
What is the value of $x + y$?

A 12 B 17 C 24 D 28 E 32

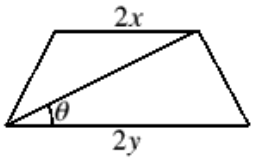
2009...

12. Which of the following has the greatest value?

A $\cos 50°$ B $\sin 50°$ C $\tan 50°$ D $\dfrac{1}{\sin 50°}$ E $\dfrac{1}{\cos 50°}$

2010...

14. The parallel sides of a trapezium have lengths $2x$ and $2y$ respectively. The diagonals are equal in length, and a diagonal makes an angle θ with the parallel sides, as shown. What is the length of each diagonal?



A $x + y$ B $\dfrac{x + y}{\sin\theta}$ C $(x+y)\cos\theta$ D $(x+y)\tan\theta$ E $\dfrac{x + y}{\cos\theta}$

2011...

7. Two sides of a triangle have lengths 4 cm and 5 cm. The third side has length x cm, where x is a positive integer. How many different values can x have?

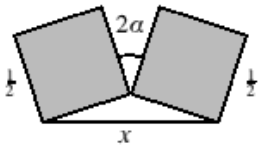
A 4 B 5 C 6 D 7 E 8

10. A triangle has two edges of length 5. What length should be chosen for the third side of the triangle so as to maximise the area within the triangle?

A 5 B 6 C $5\sqrt{2}$ D 8 E $5\sqrt{3}$

2012...

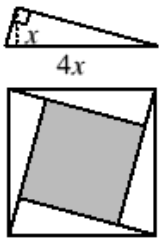
18. The diagram shows two squares, with sides of length $\frac{1}{2}$, inclined at an angle 2α to one another. What is the value of x?



A $\cos\alpha$ B $\dfrac{1}{\cos\alpha}$ C $\sin\alpha$ D $\dfrac{1}{\sin\alpha}$ E $\tan\alpha$

2013...

8. The right-angled triangle shown has a base which is 4 times its height. Four such triangles are placed so that their hypotenuses form the boundary of a large square as shown.
What is the side-length of the shaded square in the diagram?



A $2x$ B $2\sqrt{2}x$ C $3x$ D $2\sqrt{3}x$ E $\sqrt{15}x$

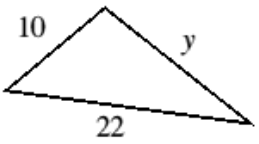
2014...

17. An oil tanker is 100 km due north of a cruise liner. The tanker sails SE at a speed of 20 kilometres per hour and the liner sails NW at a speed of 10 kilometres per hour. What is the shortest distance between the two boats during the subsequent motion?

A 100km B 80km C $50\sqrt{2}$km D 60km E $33\frac{1}{3}$km

2015…

14. The triangle shown has an area of 88 square units. What is the value of y?

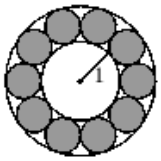


A 17.6 B $2\sqrt{46}$ C $6\sqrt{10}$ D $13\sqrt{2}$ E $8\sqrt{5}$

2016…

21. The diagram shows ten equal discs that lie between two concentric circles – an inner circle and an outer circle. Each disc touches two neighbouring discs and both circles. The inner circle has radius 1.

What is the radius of the *outer* circle?



A $2\tan 36°$ B $\frac{\sin 36°}{1-\sin 36°}$ C $\frac{1+\sin 18°}{1-\sin 18°}$ D $\frac{2}{\cos 18°}$ E $\frac{9}{5}$

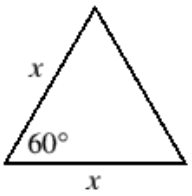
UKMT Trigonometry Answers

2005...

20. **C** Let the length in metres of the side of a pane be x. Then the area of one pane $= \frac{1}{2} \times x \times x \times \sin 60^\circ = \frac{\sqrt{3}}{4}x^2$. So

$$\frac{\sqrt{3}}{4}x^2 \approx \frac{6000}{3300}, \text{ that is } x^2 \approx \frac{4 \times 6000}{\sqrt{3} \times 3300}.$$

We conclude that $x^2 \approx \frac{7}{\sqrt{3}} \approx 4$.

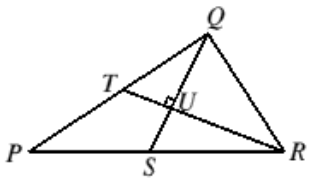


2006...

16. **C** As $\alpha + \beta = 90^\circ$, $\sin\alpha = \cos\beta$; $\cos\alpha = \sin\beta$. So $\sin\alpha\sin\beta = \sin\alpha\cos\alpha$; $\sin\alpha\cos\beta = \sin^2\alpha$; $\cos\alpha\sin\beta = \cos^2\alpha$; $\cos\alpha\cos\beta = \cos\alpha\sin\alpha$. As $\alpha < \beta$, $\alpha \neq 45^\circ$. So $\sin\alpha \neq \cos\alpha$. Thus three of the four expressions have different values.

2007...

22. **D** Let U be the point of intersection of QS and RT. As QS and RT are medians of the triangle, they intersect at a point which divides each in the ratio 2:1, so $QU = \frac{2}{3} \times 8 = \frac{16}{3}$. Therefore the area of triangle $QTR = \frac{1}{2} \times RT \times QU = \frac{1}{2} \times 12 \times \frac{16}{3} = 32$.



As a median of a triangle divides it into two triangles of equal area, the area of triangle PTR is equal to the area of triangle QTR, so the area of triangle PQR is 64.

2008...

24. **E** $1 + 3 + 5 + 7 + \ldots + (2n + 1) = (n + 1)^2$. The n in the three cases given is 12, $\frac{1}{2}(x - 1)$ and $\frac{1}{2}(y - 1)$. So, the triangle has sides of length $12 + 1$, $\frac{1}{2}(x - 1) + 1$ and $\frac{1}{2}(y - 1) + 1$. However the only right-angled triangle having sides of whole number length with hypotenuse 13 is the (5, 12, 13) triangle. So $x = 9$ and $y = 23$ (or vice versa). Hence $x + y = 32$.

2009...

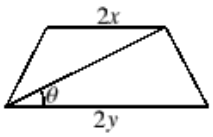
12. **E** $\cos 50^\circ < \sin 50^\circ < 1$. Hence $\dfrac{1}{\cos 50^\circ} > \dfrac{1}{\sin 50^\circ} > 1 > \sin 50^\circ > \cos 50^\circ$.

$\tan 50^\circ = \dfrac{\sin 50^\circ}{\cos 50^\circ} < \dfrac{1}{\cos 50^\circ}$ hence $\dfrac{1}{\cos 50^\circ}$ has the greatest value.

2010...

14. **E** Drop perpendiculars from the top vertices to the bottom line. The distance from the foot to the nearer base vertex is $\frac{1}{2}(2y - 2x) = y - x$. So the distance to the further base vertex is $2y - (y - x) = y + x$.

Hence $\cos\theta = \dfrac{x + y}{d}$ where d is the length of the diagonal.



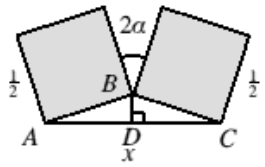
2011...

7. **D** In order to form a triangle, x must exceed the difference between 4 and 5 and x must be less than the sum of 4 and 5, i.e. $1 < x < 9$.
Hence $x = 2, 3, 4, 5, 6, 7$ or 8. So x can have 7 different values.

10. **C** The area of a triangle is $\frac{1}{2}ab \sin C$. The maximum area is attained when $\angle C = 90°$. Hence, in order to maximise the area, the triangle must be right-angled with common side lengths equal to 5. Let x be the side length of the hypotenuse, so, by Pythagoras' Theorem, $x^2 = 5^2 + 5^2 = 50$. Thus $x = 5\sqrt{2}$ is the length that should be chosen.

2012...

18. **A** In the diagram, D is the midpoint of AC. Triangle ABC is isosceles since $AB = BC = \frac{1}{2}$. Therefore, BD bisects $\angle ABC$ and BD is perpendicular to AC. The angles at a point total 360°, so $\angle ABC = 360° - 2 \times 90° - 2\alpha = 180° - 2\alpha$. Therefore $\angle ABD = \angle CBD = 90° - \alpha$. So $\angle BAD = \angle BCD = \alpha$.
Therefore $x = AC = 2 \times AD = 2 \times AB \cos\alpha = 2 \times \frac{1}{2} \cos\alpha = \cos\alpha$.

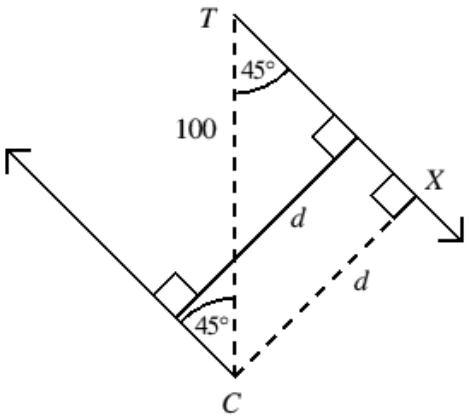


2013...

8. **B** The area of the shaded square is equal to the area of the large square minus the area of the four triangles. Thus the area of the shaded square is $(4x)^2 - 4 \times \frac{1}{2} \times 4x \times x = 16x^2 - 8x^2 = 8x^2$. So the side-length is $\sqrt{8x^2} = 2\sqrt{2}x$.

2014...

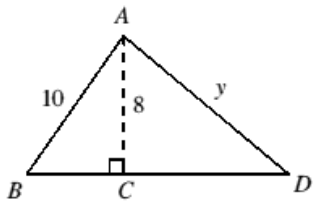
17. **C**



The tanker and the cruise liner are travelling in parallel and opposite directions, each making an angle of 45° with the line joining their starting positions. The shortest distance between the ships is d, the perpendicular distance between the parallel lines. This is independent of the speeds of the ships.
Considering triangle TCX gives $\sin 45° = \dfrac{d}{100}$
so $d = \dfrac{1}{\sqrt{2}} \times 100 = 50\sqrt{2}$.

2015...

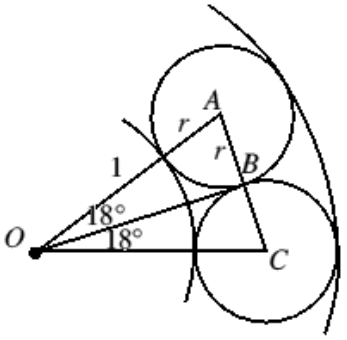
14. **E**



Let the vertices of the triangle be labelled A, B and D as shown. Let the point where the perpendicular from A meets BD be labelled C. The area of triangle ABD is given as 88. As BD is 22, AC must be 8. Considering triangle ABC and using Pythagoras' Theorem gives $BC = 6$. The remainder of the base CD is then $22 - 6 = 16$. Considering triangle ACD and using Pythagoras' Theorem again gives $y^2 = 8^2 + 16^2 = 8^2(1^2 + 2^2) = 8^2 \times 5$. So $y = 8\sqrt{5}$.

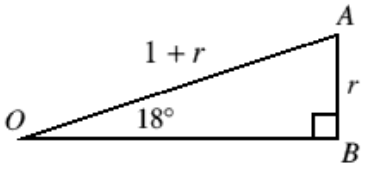
2016...

21. **C** As there are 10 discs, the adjacent lines drawn from the centre of the inner circle to the centre of each disc are separated by an angle of 36°. The line OB is a tangent to both the disc with centre A and the disc with centre C. So the points A, B and C lie on a straight line as angles OBA and OBC are both 90°.



In the second diagram, from triangle OAB we have $\sin 18° = \dfrac{r}{1+r}$ which rearranges to

$$\frac{\sin 18°}{1 - \sin 18°} = r.$$



The radius of the outer circle is

$$1 + 2r = 1 + \frac{2\sin 18°}{1 - \sin 18°} = \frac{1 + \sin 18°}{1 - \sin 18°}.$$